Linear Algebra
(Theory of Matrices)

MCQ's

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1. Find the Eigen values for the following 2×2 matrix.

A= $\begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$.

a) -3
b) 2
c) 6
d) 4

Answer: a
Explanation: We know that for any given matrix
$[A-\lambda I]X=0$ and $|A-\lambda I|=0$

$[A-\lambda I]=\begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$[A-\lambda I]=\begin{bmatrix} 1-\lambda & 8 \\ 2 & 1-\lambda \end{bmatrix}$

$|A-\lambda I|=(1-\lambda)(1-\lambda)-16=0$
$(1-\lambda)^2=16$
$(1-\lambda)=\pm 4$
$\lambda=-3$ or $\lambda=5$.

2. Find the Eigenvalue for the given matrix.

A= $\begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & 1 \\ 2 & 0 & 5 \end{bmatrix}$.

a) 13
b) -3
c) 7.1
d) 8.3

Answer: c
Explanation: We know that for any given matrix
$[A-\lambda I]X=0$ and $|A-\lambda I|=0$

$[A-\lambda I]=\begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & 1 \\ 2 & 0 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The characteristic polynomial is given by-
λ^3-(Sum of diagonal elements) λ^2+(Sum of minor of diagonal element)λ-$|A|$=0
$\lambda^3-12\lambda^2+40\lambda-39=0$
$\lambda=7.1$ or $\lambda=3$.

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3. Find the Eigen vector for value of λ=-2 for the given matrix.

A= $\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Answer: b

Explanation: We know that for any given matrix

[A-λI]X=0 and |A-λI|=0

Given that, λ=-2

[A-λI]= $\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} - (-2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

[A-λI]= $\begin{bmatrix} 3+2 & 5 \\ 3 & 1+2 \end{bmatrix}$

[A-λI]= $\begin{bmatrix} 5 & 5 \\ 3 & 3 \end{bmatrix}$

Since, [A-λI]X=0

$\begin{bmatrix} 5 & 5 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus,

5x+5y=0 and 3x+3y=0

Let x=t,

Then, y=-t

X = $\begin{bmatrix} t \\ -t \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

4. Find the Eigen vector for value of λ=3 for the given matrix.

A= $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

a) $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$

d) $\begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$

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Answer: a

Explanation: We know that for any given matrix

[A-λI]X=0 and |A-λI|=0

Given that, λ=3

[A-λI]= $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix} - (3) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[A-λI]= $\begin{bmatrix} 3-3 & 10 & 5 \\ -2 & -3-3 & -4 \\ 3 & 5 & 7-3 \end{bmatrix}$

[A-λI]= $\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix}$

Since, [A-λI]X=0

$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Row transformation

(1) Interchanging R_1 and $R_2/2$

$$\begin{bmatrix} -1 & -3 & -2 \\ 0 & 10 & 5 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(2) $R_3=R_3+3R_1$

$$\begin{bmatrix} 0 & 10 & 5 \\ 0 & 10 & 5 \\ 0 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(3) $R_2=R_2/5$ and $R_3=R_3+2R_2$

$$\begin{bmatrix} -1 & -3 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus,

-x-3y-2z=0 and 2y+z=0

Let z=t,then y= $\frac{-t}{2}$ and x= $\frac{-t}{2}$

X = $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

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5. Find the Eigen value and the Eigen Vector for the given matrix.

A= $\begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix}$.

a) 3, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

b) 9, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

c) 9, $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

d) 2, $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

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Answer: b

Explanation: We know that for any given matrix

[A-λI]X=0 and |A-λI|=0

[A-λI]= $\begin{bmatrix}3 & 4 & 2\\1 & 6 & 2\\1 & 4 & 4\end{bmatrix} - \lambda\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

The characteristic polynomial is given by-

λ^3-(Sum of diagonal elements) λ^2+(Sum of minor of diagonal element)λ-|A|=0

$\lambda^3-13\lambda^2+40\lambda-36=0$

λ=9 or λ=2

For λ=9,

[A-λI]= $\begin{bmatrix}3 & 4 & 2\\1 & 6 & 2\\1 & 4 & 4\end{bmatrix} - 9\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

[A-λI]= $\begin{bmatrix}3-9 & 4 & 2\\1 & 6-9 & 2\\1 & 4 & 4-9\end{bmatrix}$

[A-λI]= $\begin{bmatrix}-6 & 4 & 2\\1 & -3 & 2\\1 & 4 & -5\end{bmatrix}$

[A-λI]X=0

$$\begin{bmatrix}-6 & 4 & 2\\1 & -3 & 2\\1 & 4 & -5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

(1) Interchanging R_1 and R_2

$$\begin{bmatrix}1 & -3 & 2\\-6 & 4 & 2\\1 & 4 & -5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

(2) $R_2=R_2+6R_1$ and $R_3=R_3-R_1$

$$\begin{bmatrix}1 & -3 & 2\\0 & -14 & 14\\0 & 7 & -7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

(3) $R_3=R_3+R_2/2$

$$\begin{bmatrix}1 & -3 & 2\\0 & -14 & 14\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

-14y+14z=0

x-3y+2z=0

Let z=t

Then, y=t and x=t

X= $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

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6. Find the inverse of the given Matrix, using Cayley Hamilton's Theorem.

A= $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\3 & 4 & 5\end{bmatrix}$

a) $A^{-1}=\frac{1}{16}\begin{bmatrix}2 & -3 & -1\\4 & -2 & -6\\-6 & 9 & 11\end{bmatrix}$

b) $A^{-1}=\frac{1}{8}\begin{bmatrix}2 & -3 & -1\\4 & -2 & -3\\-6 & 9 & 11\end{bmatrix}$

c) $A^{-1}=\frac{1}{16}\begin{bmatrix}2 & -1 & -1\\4 & -2 & -6\\-6 & 9 & 11\end{bmatrix}$

d) $A^{-1}=\frac{1}{8}\begin{bmatrix}2 & -3 & -1\\4 & -2 & -6\\-6 & 9 & 11\end{bmatrix}$

Answer: d

Explanation: For the given Matrix,

A= $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\3 & 4 & 5\end{bmatrix}$

The characteristic polynomial is given by –

α^3-(Sum of diagonal elements) α^2+(Sum of minor of diagonal element)α-|A|=0

$\alpha^3-7\alpha^2+11\alpha-8=0$

The Cayley Hamilton's Theorem states that every matrix satisfies its Characteristic Polynomial.

Thus,

$A^3-7A^2+11A-8I=0$

To find A^{-1}, multiply both the sides of the equation by A^{-1}

$A^2 A A^{-1}-7AA A^{-1}+11A A^{-1}-8I A^{-1}=0$

We know that $A A^{-1}=I$

$A^2I-7AI+11I-8IA^{-1}=0$

$A^2-7A+11-8 A^{-1}=0$

$A^2-7A+11=8 A^{-1}$

$$8A^{-1}=\begin{bmatrix}19 & 18 & 13\\-3 & 1 & 1\\15 & 9 & 7\end{bmatrix}-7\begin{bmatrix}4 & 3 & 2\\-1 & 2 & 1\\3 & 0 & 1\end{bmatrix}+11\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$8A^{-1}=\begin{bmatrix}19-28+11 & 18-21 & 13-14\\-3+7 & 1-14+11 & 1-7\\15-21 & 9 & 7-7+11\end{bmatrix}$$

$$8A^{-1}=\begin{bmatrix}2 & -3 & -1\\4 & -2 & -6\\-6 & 9 & 11\end{bmatrix}$$

$$A^{-1}=\frac{1}{8}\begin{bmatrix}2 & -3 & -1\\4 & -2 & -6\\-6 & 9 & 11\end{bmatrix}.$$

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7. Find the value of A3 where A= $\begin{bmatrix} -1 & -1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} 3 & 5 & -1 \\ -2 & -9 & 2 \\ -2 & -4 & -5 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 5 & -1 \\ 1 & -9 & 1 \\ -2 & -4 & -5 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 5 & -1 \\ -2 & -9 & 1 \\ -2 & -4 & -5 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 5 & -1 \\ -1 & -9 & 1 \\ -2 & -4 & -5 \end{bmatrix}$

Answer: c

Explanation: For the given Matrix,

A= $\begin{bmatrix} -1 & -1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$

The characteristic polynomial is given by-

α^3-(Sum of diagonal elements) α^2+(Sum of minor of diagonal element)α-|A|=0

$\alpha^3-\alpha^2+3\alpha+5=0$

The Cayley Hamilton's Theorem states that every matrix satisfies its Characteristic Polynomial.

Thus,

$A^3-A^2+3A+5I=0$

$A^3=A^2-3A-5I$

$A^3= \begin{bmatrix} 5 & 2 & 5 \\ -2 & -1 & -2 \\ 4 & 2 & 3 \end{bmatrix} - 3 \begin{bmatrix} -1 & -1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3= \begin{bmatrix} 5+3-5 & 2+3 & 5-6 \\ -2+0 & -1-3-5 & -2+3 \\ 4-6 & 2-6 & 3-3-5 \end{bmatrix}$

$A^3= \begin{bmatrix} 3 & 5 & -1 \\ -2 & -9 & 1 \\ -2 & -4 & -5 \end{bmatrix}$.

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8. Find the value of A^3+19A, A= $\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix}$.

a) $\begin{bmatrix} 42 & -14 & 70 \\ 21 & +21 & -21 \\ 105 & 119 & 203 \end{bmatrix}$

b) $\begin{bmatrix} 42 & -7 & 70 \\ 21 & -21 & -21 \\ 105 & 119 & 203 \end{bmatrix}$

c) $\begin{bmatrix} 42 & -14 & 70 \\ 21 & -21 & -21 \\ 105 & 119 & 203 \end{bmatrix}$

d) $\begin{bmatrix} 42 & -7 & 70 \\ 21 & +21 & -21 \\ 105 & 119 & 203 \end{bmatrix}$

Answer: c

Explanation: Explanation: For the given Matrix,

A= $\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix}$

The characteristic polynomial is given by –

α^3-(Sum of diagonal elements) α^2+(Sum of minor of diagonal element)α-|A|=0

$\alpha^3-7\alpha^2+19\alpha-49=0$

The Cayley Hamilton's Theorem states that every matrix satisfies its Characteristic Polynomial.

Thus,

$A^3-7A^2+19A-49I=0$

$A^3+19A=7A^2+49I$

$A^3+19A=7\begin{bmatrix} -1 & -2 & 10 \\ 3 & -10 & -3 \\ 15 & 17 & 22 \end{bmatrix} + 49\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3+19A=\begin{bmatrix} -7+49 & -17 & 70 \\ 21 & -70+49 & -21 \\ 105 & 119 & 154+49 \end{bmatrix}$

$A^3+19A=\begin{bmatrix} 42 & -14 & 70 \\ 21 & -21 & -21 \\ 105 & 119 & 203 \end{bmatrix}$.

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9. Find the value of 2A^3+4A^2, where = $\begin{bmatrix} 5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} -200 & 0 & -24 \\ 24 & -32 & -24 \\ -72 & 96 & -56 \end{bmatrix}$

b) $\begin{bmatrix} -200 & 0 & -24 \\ 24 & -32 & -12 \\ -72 & 96 & -56 \end{bmatrix}$

c) $\begin{bmatrix} -200 & 0 & -24 \\ 12 & -32 & -24 \\ -72 & 96 & -56 \end{bmatrix}$

d) $\begin{bmatrix} -100 & 0 & -12 \\ 12 & -16 & -12 \\ -36 & 48 & -28 \end{bmatrix}$

Answer: a

Explanation: Explanation: For the given Matrix,

A= $\begin{bmatrix} 5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix}$

The characteristic polynomial is given by-

α^3-(Sum of diagonal elements) α^2+(Sum of minor of diagonal element)α-|A|=0

$\alpha^3+2\alpha^2-12\alpha-40=0$

The Cayley Hamilton's Theorem states that every matrix satisfies its Characteristic Polynomial.

Thus,

$A^3+2A^2-12A+40I=0$

$A^3+2A^2=12A-40I$

$A^3+2A^2=12\begin{bmatrix} 5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3+2A^2=\begin{bmatrix} -60-40 & 0 & -12 \\ 12 & 24-40 & -12 \\ -36 & 48 & 12-40 \end{bmatrix}$

$A^3+2A^2=\begin{bmatrix} -100 & 0 & -12 \\ 12 & -16 & -12 \\ -36 & 48 & -28 \end{bmatrix}$

$2A^3+4A^2=\begin{bmatrix} -200 & 0 & -24 \\ 24 & -32 & -24 \\ -72 & 96 & -56 \end{bmatrix}$.

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10. Find the value of A^3-3A^2-28A, A = $\begin{bmatrix} -1 & 2 & 8 \\ -2 & 3 & 0 \\ -4 & 5 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} 80 & -126 & -504 \\ 126 & -172 & -63 \\ 252 & -316 & -46 \end{bmatrix}$

b) $\begin{bmatrix} 80 & -126 & -504 \\ 126 & -172 & -63 \\ 252 & -315 & -46 \end{bmatrix}$

c) $\begin{bmatrix} 40 & -126 & -504 \\ 126 & -172 & -63 \\ 252 & -315 & -46 \end{bmatrix}$

d) $\begin{bmatrix} 40 & -126 & -504 \\ 126 & -172 & -63 \\ 252 & -316 & -46 \end{bmatrix}$

Answer: b

Explanation: For the given Matrix,

A= $\begin{bmatrix} -1 & 2 & 8 \\ -2 & 3 & 0 \\ -4 & 5 & 1 \end{bmatrix}$

The characteristic polynomial is given by-

α^3-(Sum of diagonal elements) α^2+(Sum of minor of diagonal element)α-|A|=0

$\alpha^3-3\alpha^2+35\alpha-17=0$

The Cayley Hamilton's Theorem states that every matrix satisfies its Characteristic Polynomial.

Thus,

$A^3-3A^2+35A-17I=0$

On performing long division $(\alpha^3-3\alpha^2+35\alpha-17)/(\alpha^2-7\alpha)$

$Q=\alpha+4$ and $R=63\alpha-17$

Using division properties,

$\alpha^3-3\alpha^2+35\alpha-17=(\alpha^2-7\alpha)\times(\alpha+4)+(63\alpha-17)$

$\alpha^3-3\alpha^2+35\alpha-17=(\alpha^3-3\alpha^2-28\alpha)+(63\alpha-17)$

$0=(\alpha^3-3\alpha^2-28\alpha)+(63\alpha-17)$ ——————— (From Characteristic Polynomial)

$(\alpha^3-3\alpha^2-28\alpha) = -63\alpha+17$

$(A^3-3A^2-28A) = -63A+17I$

$(A^3-3A^2-28A) = -63\begin{bmatrix} -1 & 2 & 8 \\ -2 & 3 & 0 \\ -4 & 5 & 1 \end{bmatrix} + 17\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$(A^3-3A^2-28A)=\begin{bmatrix} 63+17 & -126 & -504 \\ 126 & 17-189 & -63 \\ 252 & -315 & 17-63 \end{bmatrix}$

$(A^3-3A^2-28A)=\begin{bmatrix} 80 & -126 & -504 \\ 126 & -172 & -63 \\ 252 & -315 & -46 \end{bmatrix}$.

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11. Which of the following is not a necessary condition for a matrix, say A, to be diagonalizable?
a) A must have n linearly independent eigen vectors
b) All the eigen values of A must be distinct
c) A can be an idempotent matrix
d) A must have n linearly dependent eigen vectors
Answer: d
Explanation: The theorem of diagonalization states that, 'An n×n matrix A is diagonalizable, if and only if, A has n linearly independent eigenvectors.' Therefore, if A has n distinct eigen values, say $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$, then the corresponding eigen vectors are said to be linearly independent. Also, all idempotent matrices are said to be diagonalizable.

12. The geometric multiplicity of λ is its multiplicity as a root of the characteristic polynomial of A, where λ be the eigen value of A.
a) True
b) False
Answer: b
Explanation: The diagonalization theorem in terms of multiplicities of eigen values is defined as follows,
The algebraic multiplicity of λ is its multiplicity as a root of the characteristic polynomial of A.
The geometric multiplicity of λ is the dimension of the λ-eigenspace.

13. If A is diagonalizable then, ___________
a) $A^n = (PDP^{-1})^n = PD^nP^n$
b) $A^n = (PDP^{-1})^n = PD^nP^1$
c) $A^n = (PDP^{-1})^n = PD^nP^{-1}$
d) $A^n = (PDP^{-1})^n = PD^nP$
Answer: c
Explanation: The definition of diagonalization states that, An n × n matrix A is diagonalizable if there exists an n × n invertible matrix P and an n × n diagonal matrix D such that,
$P^{-1} AP = D$
$A = PDP^{-1}$
$A^n = (PDP^{-1})^n = PD^nP^{-1}$

14. The computation of power of a matrix becomes faster if it is diagonalizable.
a) True
b) False
Answer: a
Explanation: Some of the applications of diagonalization of a matrix are:
The powers of a diagonalized matrix can be computed easily since the result is nothing but the powers of the diagonal elements obtained by diagonalization.
Reducing quadratic forms to canonical forms by orthogonal transformations.
In mechanics, it can be used to find the natural frequency of vibrations.

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15. Find the invertible matrix P, by using diagonalization method for the following matrix.

A = $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

a) A = $\begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

b) A = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

c) A = $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

d) A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

Answer: b

Explanation: Procedure to find the invertible matrix is as follows,

Step 1: Find the eigen values of the given matrix.

A = $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 2-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{vmatrix} = 0$ .... (i)

$(2-\lambda)((2-\lambda)(1-\lambda)) = 0$

$(2-\lambda)^2(1-\lambda) = 0$

$\lambda = 2, 2, 1$

Step 2: Compute the eigen vectors

Consider $\lambda = 2$,

$(A - \lambda I)\vec{X} = \vec{0}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \vec{X} = \vec{0} \xrightarrow{Reducing\ further,\ weget,} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \vec{X} = \vec{0}$$

x2 is the free variable, hence, x2 = s

Let x1 = -t, x3 = t, since x1+x3 = 0

$\vec{X} = s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\vec{X_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \vec{X_2} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Consider $\lambda = 1$

$(A - \lambda I)\vec{X} = \vec{0}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \vec{X} = \vec{0} \xrightarrow{Reducing\ further,\ weget,} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \vec{X} = \vec{0}$$

x1 = 0, Let x2 = -s and x3 = s since x2+x3=0

$\vec{X} = s \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$\vec{X_3} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Step 3: Formation of the invertible matrix.

$P = [\vec{X_1}\vec{X_2}\vec{X_3}]$

P = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

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16. Determine the algebraic and geometric multiplicity of the following matrix. $\begin{bmatrix} 2 & 4 & -4 \\ 0 & 4 & 2 \\ -2 & 4 & 4 \end{bmatrix}$

a) Algebraic multiplicity = 1, Geometric multiplicity = 2

b) Algebraic multiplicity = 1, Geometric multiplicity = 3

c) Algebraic multiplicity = 2, Geometric multiplicity = 2

d) Algebraic multiplicity = 2, Geometric multiplicity = 1

Answer: d

Explanation: The eigen values of the given matrix can be computed as,

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 3 & 3-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)((3-\lambda)(1-\lambda)) = 0$

$(1-\lambda)^2(3-\lambda) = 0$

$\lambda = 1, 1, 3$ are the eigen values of the matrix. So, the algebraic multiplicity of $\lambda = 1$ is two.

For $\lambda = 3$,

$(A - \lambda I)\vec{X} = \vec{0}$

$$\begin{bmatrix} -2 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix} \vec{X} = \vec{0} \xrightarrow{\text{Reducing further, we get,}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \vec{X} = \vec{0}$$

$x1 = 0, x3 = 0$

$x2$ is the free variable, therefore let $x2 = s$,

Hence, $\vec{X_1} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

For $\lambda = 1$,

$(A - \lambda I)\vec{X} = \vec{0}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{X} = \vec{0} \xrightarrow{\text{Reducing further, we get,}} \begin{bmatrix} 1 & 2/3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \vec{X} = \vec{0}$$

$x1 + \frac{2}{3}x2 = 0, x3 = 0$

Let $x1 = -2$ and $x3 = 3$,

$$\vec{X_2} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$

Thus, there corresponds only one eigen vector for the repeated eigen value $\lambda=1$. Thus, the geometric multiplicity of $\lambda = 2$ is one.

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17. Given P = $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$ $and\, D = \begin{bmatrix} 6 & 0 \\ 0 & -1 \end{bmatrix}$, find A^3.

a) $\begin{bmatrix} 61 & 62 \\ 156 & 154 \end{bmatrix}$

b) $\begin{bmatrix} 61 & 62 \\ 155 & 154 \end{bmatrix}$

c) $\begin{bmatrix} 61 & 60 \\ 155 & 154 \end{bmatrix}$

d) $\begin{bmatrix} 61 & 62 \\ 155 & 150 \end{bmatrix}$

Answer: b

Explanation: From the theory of diagonalization, we know that,

$A = PDP^{-1}$

$A^n = PD^nP^{-1}$

Given, P= $\begin{bmatrix} 2 & -1 \\ 5 & 1, \end{bmatrix}$ $hence P^{-1} = \frac{1}{7}\begin{bmatrix} 1 & 1 \\ -5 & 2 \end{bmatrix}$

Therefore, $A^3 = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 6 & 0 \\ 0 & -1 \end{bmatrix}^3\begin{bmatrix} 1 & 1 \\ -5 & 2 \end{bmatrix}$................................ since n=3

$A^3 = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 216 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -5 & 2 \end{bmatrix}$

$A^3 = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 216 & 216 \\ 5 & -2 \end{bmatrix}$

$A^3 = \frac{1}{7}\begin{bmatrix} 427 & 434 \\ 1085 & 1078 \end{bmatrix}$

$A^3 = \begin{bmatrix} 61 & 62 \\ 155 & 154 \end{bmatrix}$

18. Find the trace of the matrix A= $\begin{bmatrix} 1 & 0 & 6 \\ 0 & 5 & 0 \\ 0 & 4 & 4 \end{bmatrix}$

a) 0

b) 10

c) 4

d) 1

Answer: b

Explanation: The sum of the entries on the main diagonal is called the trace of matrix A.

Therefore, trace = 1+5+4 = 9.

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19. The determinant of the matrix whose eigen values are 4, 2, 3 is given by, ______

a) 9
b) 24
c) 5
d) 3
Answer: b
Explanation: The product of the eigen values of a matrix gives the determinant of the matrix,
Therefore, Δ = 24.

20. Which of the following relation is correct?

a) $A = A_T$
b) $A = -A_T$
c) $A = A_{2T}$
d) $A = A_{T/2}$

Answer: a
Explanation: To prove that $A = A^T$, let us consider an example,

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(3-\lambda) = 0$
$3 - \lambda - 3\lambda + \lambda^2 = 0$
$\lambda^2 - 4\lambda + 3 = 0$
$(\lambda - 3)(\lambda - 1) = 0$
$\lambda = 1, 3$

Consider $A^T = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$(1-\lambda)(3-\lambda) = 0$, which is similar to the result obtained for A, hence the eigen values are same.

21. Who introduced the term matrix?

a) James Sylvester
b) Arthur Cayley
c) Girolamo Cardano
d) Paul Erdos
Answer: a
Explanation: The term 'matrix' was introduced by James Sylvester during the 19th century. Later in 1850, Arthur Cayley developed the algebraic aspect of matrices in two of his papers.

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22. Which among the following is listed under the law of transposes?

a) $(A^T)^T = A^I$
b) (A*)* = A*
c) $(cA)^T = cA^T$
d) $(AB)^T = A^TB^T$

Answer: c
Explanation: The following are listed under the law of transposes:
(A*)* = A
$(A^T)^T = A$
$(AB)^T = B^T A^T$
$(cA)^T = cA^T$
$(A \pm B)^T = A^T \pm B^T$ (and for $*$)
If A is symmetric, $A = A^T$

23. The matrix which remains unchanged under transposition is known as skew symmetric matrix.

a) False
b) True

Answer: a
Explanation: The matrix which remains unchanged under transposition is known as symmetric matrix.

For example, if we consider a symmetric matrix A = $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 7 \end{bmatrix}$ and take the transpose of it, we get,

$A^T = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 7 \end{bmatrix}$

24. Find the values of x and y in the matrix below if the matrix is a skew symmetric matrix.

P = $\begin{bmatrix} 0 & y & -4 \\ -5 & 0 & 8 \\ x+y & -8 & 0 \end{bmatrix}$

a) x = -1, y = 5
b) x = -9, y = -5
c) x = 9, y = 5
d) x = 1, y = -5

Answer: a
Explanation: The general form of a skew symmetric matrix is given by,

$\begin{bmatrix} 0 & w1 & -w2 \\ -w1 & 0 & w3 \\ w2 & -w3 & 0 \end{bmatrix}$ Therefore, from the given matrix,

y = 5,
$x + y = 4 \rightarrow x + 5 = 4 \rightarrow x = -1$

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25. Which of the following is known as the reversal rule?

a) $(AB)^{-1} = B^{-1}A^{-1}$
b) $(AB)^{-1} = A^{-1} B^{-1}$
c) $(BA)^{-1} = B^{-1} A^{-1}$
d) $(BA)^{-1} = B^{-1} A$

Answer: a

Explanation: The reversal rule of matrix multiplication states that, 'the inverse of the product of two matrices is equal to the product of their individual inverses, taken in the reverse order'.

26. Every Identity matrix is an orthogonal matrix.

a) True
b) False

Answer: b

Explanation: An orthogonal matrix can be defined as 'a matrix having its entries as orthogonal unit vectors' or it can also be defined as 'a matrix whose transpose is equal to its inverse'. Since this property is satisfied by an identity matrix, every identity matrix is an orthogonal matrix.

27. Which of the following matrix is orthogonal?

a) $\begin{bmatrix} 0.33 & 0.67 & -0.67 \\ 0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{bmatrix}$

b) $\begin{bmatrix} 0.33 & 0.67 & -0.67 \\ -0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{bmatrix}$

c) $\begin{bmatrix} 0.33 & 0.67 & -0.67 \\ -0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{bmatrix}$

d) $\begin{bmatrix} 0.33 & 0.67 & -0.67 \\ -0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{bmatrix}$

Answer: b

Explanation: Out of the given options, $\begin{bmatrix} 0.33 & 0.67 & -0.67 \\ -0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{bmatrix}$ satisfies the condition for orthogonality, i.e. $AA^T = I$

$$\begin{bmatrix} 0.33 & 0.67 & -0.67 \\ -0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{bmatrix} \begin{bmatrix} 0.33 & -0.67 & 0.67 \\ 0.67 & 0.67 & 0.33 \\ -0.67 & 0.33 & 0.67 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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28. Find the symmetric matrix for the given quadratic form,

$$Q = 2x_1^2 + 2x_2^2 + 5x_3^2 - 3x_1x_2 + 7x_3x_1.$$

a) $\begin{bmatrix} 1 & \frac{-7}{2} & \frac{3}{2} \\ \frac{-7}{2} & 2 & 0 \\ \frac{3}{2} & 0 & \frac{5}{2} \end{bmatrix}$

b) $\begin{bmatrix} 1 & \frac{-3}{2} & \frac{7}{2} \\ \frac{-3}{2} & 2 & 0 \\ \frac{7}{2} & 0 & \frac{5}{2} \end{bmatrix}$

c) $\begin{bmatrix} 1 & \frac{3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 2 & 0 \\ -\frac{7}{2} & 0 & \frac{5}{2} \end{bmatrix}$

d) $\begin{bmatrix} 1 & \frac{-3}{2} & \frac{7}{2} \\ \frac{-3}{2} & 2 & 0 \\ \frac{7}{2} & 0 & 5 \end{bmatrix}$

Answer: b

Explanation: The following steps need to be followed to obtain the symmetric matrix:

Step 1: There are three variables in the given quadratic equation. Hence, the symmetric matrix to be formed should be of dimension 3×3 and the general form can be written as,

Q = $\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$

Step 2: Place the square term coefficients of the quadratic equation (2, 2, 5) on the diagonal of the matrix.

Q = $\begin{bmatrix} 2 & c_{12} & c_{13} \\ c_{21} & 2 & c_{23} \\ c_{31} & c_{32} & 5 \end{bmatrix}$

Step 3: Place the remaining coefficients of $x_i x_j \, at \, c_{ij}$, i.e. coefficient of x_1 x_2 (-3) at c_{12} and so on.

Q = $\begin{bmatrix} 2 & -3 & 0 \\ 0 & 2 & 0 \\ 7 & 0 & 5 \end{bmatrix}$

Step 4: For a symmetric matrix, S = $\frac{1}{2}(Q + Q^T)$

S = $\frac{1}{2} \begin{bmatrix} 2 & -3 & 0 \\ 0 & 2 & 0 \\ 7 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 7 \\ -3 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$S = \frac{1}{2} \begin{bmatrix} 2 & -3 & 7 \\ -3 & 4 & 0 \\ 7 & 0 & 10 \end{bmatrix}$$

S = $\begin{bmatrix} 1 & \frac{-3}{2} & \frac{7}{2} \\ \frac{-3}{2} & 2 & 0 \\ \frac{7}{2} & 0 & \frac{5}{2} \end{bmatrix}$

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29. Which of the following is known as Hadamard matrix?

a) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Answer: b

Explanation: Hadamard matrix is named after a famous French mathematician, Jacques Hadamard. It is defined as 'a square matrix whose entries are only 1 or -1 and whose column (or row) vectors orthogonal'.

30. The sum of two skew-symmetric matrices is also a skew-symmetric matrix.

a) False

b) True

Answer: b

Explanation: To prove the above statement, let us consider an example,

$A = \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$

Therefore, $A + A = \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 & 2 \\ 8 & 0 & -9 \\ -2 & 6 & 0 \end{bmatrix}$ which is also a skew-symmetric matrix.

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