Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (New) EXAMINATION – WINTER 2019

Subject Code: 3130005 **Date: 26/11/2019**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the real and imaginary parts of $f(z) = \frac{3i}{2+3i}$.	03
	(b)	State De-Movire's formula and hence evaluate $\left(1+i\sqrt{3}\right)^{100} + \left(1-i\sqrt{3}\right)^{100}$.	04
	(c)	Define harmonic function. Show that $u(x,y) = \sinh x \sin y$ is harmonic function, find its harmonic conjugate $v(x,y)$.	07
Q.2	**(a)**	Determine the Mobius transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ into $w_1 = -1,\ w_2 = -i,\ w_3 = 1$.	03
	(b)	Define $log z$, prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$.	04
	(c)	Expand $f(z) = \frac{1}{(z-1)(z+2)}$ valid for the region (i) $\lvert z\rvert < 1$ (ii) $1 < \lvert z\rvert < 2$ (iii) $\lvert z\rvert > 2$.	07
		OR	
	(c)	Find the image of the infinite strips (i) $\frac{1}{4} \le y \le \frac{1}{2}$ (ii) $0 < y < \frac{1}{2}$ under the transformation $= \frac{1}{z}$. Show the region graphically.	07
Q.3	**(a)**	Evaluate $\int_c (x - y + ix^2)dz$ where c is a straight line from $z = 0$ to $z = 1 + i$.	03
	(b)	Check whether the following functions are analytic or not at any point, (i) $f(z) = 3x + y + i(3y - x)$ (ii) $f(z) = z^{3/2}$.	04
	(c)	Using residue theorem, evaluate $\int_0^\infty \frac{dx}{(x^2+1)^2}$.	07
		OR	
Q.3	**(a)**	Expand Laurent series of $f(z) = \frac{1-e^z}{z}$ at $z = 0$ and identify the singularity.	03
	(b)	If $f(z) = u + iv$, is an analytic function , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert Re f(z)\rvert^2 = 2\lvert f'(z)\rvert^2$.	04
	(c)	Evaluate the following: i. $\int_c \frac{z+3}{z-1}\, dz$ where c is the circle (a) $\lvert z\rvert = 2$ (b) $\lvert z\rvert = \frac{1}{2}$. ii. $\int_c \frac{\sin z}{\left(z-\frac{\pi}{4}\right)^3}\, dz$ where c is the circle $\lvert z\rvert = 1$.	07

Q.4 **(a)** Evaluate $\int_0^{2+4i} Re(z)dz$ along the curve $z(t) = t + it^2$. **03**

(b) Solve $x^2p + y^2q = (x+y)z$. **04**

(c) Solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for the condition of heat along rod without radiation subject to the conditions (i) $\frac{\partial u}{\partial t} = 0$ for $x = 0$ and $x = l$;
(ii) $u = lx - x^2$ at $t = 0$ for all x . **07**

OR

Q.4 **(a)** Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{2x+3y}$. **03**

(b) Solve $px + qy = pq$ using Charpit's method. **04**

(c) Find the general solution of partial differential equation $u_{xx} = 9u_y$ using method of separation of variables. **07**

Q.5 **(a)** Using method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$. **03**

(b) Solve $z(xp - yq) = y^2 - x^2$. **04**

(c) A string of length $L = \pi$ has its ends fixed at $x = 0$ and $x = \pi$. At time $t = 0$, the string is given a shape defined by $f(x) = 50x(\pi - x)$, then it is released. Find the deflection of the string at any time t. **07**

OR

Q.5 **(a)** Solve $p^3 + q^3 = x + y$. **03**

(b) Find the temperature in the thin metal rod of length l with both the ends insulated and initial temperature is $\sin {\pi x}/{l}$. **04**

(c) Derive the one dimensional wave equation that governs small vibration of an elastic string . Also state physical assumptions that you make for the system. **07**

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