

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE - SEMESTER-I & II (NEW) EXAMINATION – SUMMER-2019****Subject Code: 3110014****Date: 06/06/2019****Subject Name: Mathematics – I****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- |  | <b>Marks</b> |
|--|--------------|
| <b>Q.1</b> (a) Use L'Hospital's rule to find the limit of $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right)$ .  | <b>03</b>    |
| (b) Define Gamma function and evaluate $\int_0^{\infty} e^{-x^2} dx$ .   | <b>04</b>    |
| (c) Evaluate $\int_0^3 \int_{\sqrt{x}}^1 e^{y^2} dy dx$ .  | <b>07</b>    |
| <br>   |              |
| <b>Q.2</b> (a) Define the convergence of a sequence $(a_n)$ and verify whether the sequence whose $n^{th}$ term is $a_n = \left( \frac{n+1}{n-1} \right)^n$ converges or not.  | <b>03</b>    |
| (b) Sketch the region of integration and evaluate the integral $\iint_R (y - 2x^2) dA$ where $R$ is the region inside the square $ x  +  y  = 1$ .   | <b>04</b>    |
| (c) (i) Find the sum of the series $\sum_{n \geq 2} \frac{1}{4^n}$ and $\sum_{n \geq 1} \frac{4}{(4n-3)(4n+1)}$ .  | <b>07</b>    |
| (ii) Use Taylor's series to estimate $\sin 38^\circ$ .   |              |
| <b>OR</b>  |              |
| (c) Evaluate the integrals $\int_0^{\infty} \int_0^{\infty} \frac{1}{(1+x^2+y^2)^2} dx dy$ and $\int_0^1 \int_{\sqrt{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2} dx dy dz$ .  | <b>07</b>    |
| <br>   |              |
| <b>Q.3</b> (a) If an electrostatic field $E$ acts on a liquid or a gaseous polar dielectric, the net dipole moment $P$ per unit volume is $P(E) = \frac{e^E + e^{-E}}{e^E - e^{-E}} - \frac{1}{E}$ . Show that $\lim_{E \rightarrow 0^+} P(E) = 0$ . | <b>03</b>    |
| (b) For what values of the constant $k$ does the second derivative test guarantee that $f(x, y) = x^2 + kxy + y^2$ will have a saddle point at $(0,0)$ ? A local minimum at $(0,0)$ ?  | <b>04</b>    |
| (c) Find the series radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n}$ . For what values of $x$ does the series converge absolutely?  | <b>07</b>    |
| <b>OR</b>  |              |
| <b>Q.3</b> (a) Determine whether the integral $\int_0^3 \frac{dx}{x-1}$ converges or diverges.   | <b>03</b>    |
| (b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$ .   | <b>04</b>    |

- (c) Check the convergence of the series  $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^3}$  and  $\sum_{n=0}^{\infty} (-1)^n (\sqrt{n+1} + \sqrt{n} - \sqrt{n})$ . 07

- Q.4 (a) Show that the function  $f(x, y) = \frac{2x^2y}{x^4+y^2}$  has no limit as  $(x, y)$  approaches to  $(0,0)$ . 03

- (b) Suppose  $f$  is a differentiable function of  $x$  and  $y$  and  $g(u, v) = f(e^u + \sin v, e^u + \cos v)$ . Use the following table to calculate  $g_u(0,0), g_v(0,0), g_u(1,2)$  and  $g_v(1,2)$ . 04

	$f$	$g$	$f_x$	$f_y$
$(0,0)$	3	6	4	8
$(1,2)$	6	3	2	5

- (c) Find the Fourier series of  $2\pi$ -periodic function  $f(x) = x^2, 0 < x < 2\pi$  and hence deduce that  $\frac{\pi^2}{6} = \sum_{n=0}^{\infty} \frac{1}{n^2}$ . 07

OR

- Q.4 (a) Verify that the function  $u = e^{-\alpha^2 k^2 t} \cdot \sin kx$  is a solution of the heat conduction equation  $u_t = \alpha^2 u_{xx}$ . 03

- (b) Find the half-range cosine series of the function  $f(x) = \begin{cases} 2, & -2 < x < 0 \\ 0, & 0 < x < 2 \end{cases}$ . 04

- (c) Find the points on the sphere  $x^2 + y^2 + z^2 = 4$  that are closest to and farthest from the point  $(3,1,-1)$ . 07

- Q.5 (a) Find the directional derivative  $D_u f(x, y)$  if  $f(x, y) = x^3 - 3xy + 4y^2$  and  $u$  is the unit vector given by angle  $\theta = \frac{\pi}{6}$ . What is  $D_u f(1,2)$ ? 03

- (b) Find the area of the region bounded by the curves  $y = \sin x, y = \cos x$  and the lines  $x = 0$  and  $x = \frac{\pi}{4}$ . 04

- (c) Prove that  $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$  is diagonalizable and use it to find  $A^{13}$ . 07

OR

- Q.5 (a) Define the rank of a matrix and find the rank of the matrix  $A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$ . 03

- (b) Use Gauss-Jordan algorithm to solve the system of linear equations  $2x_1 + 2x_2 - x_3 + x_5 = 0$   
 $-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$   
 $x_1 + x_2 - 2x_3 - x_5 = 0$   
 $x_3 + x_4 + x_5 = 0$  04

- (c) State Cayley-Hamilton theorem and verify it for the matrix  $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ . 07