Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110015 **Date: 01/01/2020**

Subject Name: Mathematics –2

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the length of curve of the portion of the circular helix $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$ from $t = 0$ to $t = \pi$	**03**
	(b)	$\int_{(1,2)}^{(3,4)} \left(xy^2 + y^3\right)dx + \left(x^2y + 3xy^2\right)dy$ is independent of path joining the points (1, 2) and (3,4). Hence, evaluate the integral.	**04**
	(c)	Verify tangential form of Green's theorem for $\vec{F} = \left(x - \sin y\right)\hat{i} + \left(\cos y\right)\hat{j}$, where C is the boundary of the region bounded by the lines $y = 0, x = \pi/2$ and $y = x$.	**07**
Q.2	**(a)**	Find the Laplace transform of $f(t)$ defined as $f(t) = \frac{t}{k}$, $0 < t < k$; $= 1$, $t > k$	**03**
	(b)	Find the inverse Laplace transform of $\frac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$	**04**
	(c)	(i) Calculate the curl of the vector $xyz\,\hat{i} + 3x^2y\,\hat{j} + \left(xz^2 - y^2z\right)\hat{k}$ (ii) The temperature at any point in space is given by $T = xy + yz + zx$. Determine the derivative of T in the direction of the vector $3\hat{i} - 4\hat{k}$ at the point (1, 1, 1).	**07**
		OR	
	(c)	Let $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$, $r = \lvert\bar{r}\rvert$, and $\bar{a}$ is a constant vector. Find the value of $div\left(\frac{\bar{a} \times \bar{r}}{r^n}\right)$	**07**
Q.3	**(a)**	Find constants a, b and c such that $\vec{V} = \left(x + 2y + a\,z\right)\hat{i} + \left(bx - 3y - z\right)\hat{j} + \left(4x + cy + 2z\right)\hat{k}$ is irrotational.	**03**
	(b)	Using Fourier cosine integral representation show that $\int_0^\infty \frac{\cos \omega x}{k^2 + \omega^2}\,d\omega = \frac{\pi e^{-kx}}{2k}$	**04**
	(c)	Solve the following differential equations: (i) $\cos(x + y)\,dy = dx$ (ii) $\sec^2 y\,\frac{dy}{dx} + x\tan y = x^3$	**07**
		OR	

Q.3	**(a)**	Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t}\,dt$ (ii) $t^2 u(t-3)$	**03**
	(b)	Using Convolution theorem obtain $L^{-1}\left(\frac{1}{s\left(s^2+a^2\right)}\right)$	**04**
	(c)	Find the power series solution of $\frac{d^2y}{dx^2}+xy=0$	**07**
Q.4	**(a)**	Find the Laplace transform of the waveform $f(t)=\left(\frac{2t}{3}\right), 0\le t\le 3$	**03**
	(b)	Using the Laplace transforms, find the solution of the initial value problem $y''+25y=10\cos 5t \quad y(0)=2, y'(0)=0$	**04**
	(c)	Using variation of parameter method solve $\left(D^2+1\right)y=x\sin x$	**07**
		OR	
Q.4	**(a)**	Solve $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}$	**03**
	(b)	Solve $y'''-3y''+3y'-y=4e^t$	**04**
	(c)	Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=2x^2+3e^{-x}$ using method of undetermined coefficients.	**07**
Q.5	**(a)**	Classify the singular points of the equation $x^3(x-2)y''+x^3y'+6y=0$	**03**
	(b)	Solve $\left(D^2+4\right)y=\cos 2x$	**04**
	(c)	Solve (i) $ye^x dx+\left(2y+e^x\right)dy=0$ (ii) $\frac{dy}{dx}+2y\tan x=\sin x$	**07**
		OR	
Q.5	**(a)**	Solve $\frac{dy}{dx}=\frac{y^3}{e^{2x}+y^2}$	**03**
	(b)	If $y_1=x$ is one of solution of $x^2y''+xy'-y=0$ find the second solution.	**04**
	(c)	Using Frobenius method solve $x^2y''+4xy'+\left(x^2+2\right)y=0$	**07**

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