Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110014 **Date: 17/01/2020**

Subject Name: Mathematics – I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Find the equations of the tenagent plane and normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1)	**03**
	(b)	Evaluate $\lim_{x\to 0} \frac{xe^x - log(1+x)}{x^2}$	**04**
	(c)	Using Gauss Elimination method solve the following system -x+3y+4z =30 3x+2y-z =9 2x-y+2z =10	**07**
Q.2	**(a)**	Test the convergence of the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \cdots + \left(\frac{n}{2n+1}\right)^n + \cdots$	**03**
	(b)	Discuss the Maxima and Minima of the function $3x^2 - y^2 + x^3$	**04**
	(c)	Find the fourier series of $f(x) = \frac{(\pi - x)}{2}$ in the interval (0,2π)	**07**
		OR	
	(c)	Change the order of integration and evaluate $\int_0^1 \int_x^1 siny^2\, dy\, dx$	**07**
Q.3	**(a)**	Find the value of $\beta\left(\frac{7}{2}, \frac{5}{2}\right)$	**03**
	(b)	Obtain the fourier cosine series of the function f(x) = e^x in the range (0,l)	**04**
	(c)	Find the maximum and minimum distance fom the point (1,2,2) to the sphere $x^2 + y^2 + z^2 = 36$	**07**
		OR	
Q.3	**(a)**	Test the convergence of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots.$	**03**
	(b)	Evaluate $\iint (x^2 - y^2)\, dx\, dy$ over the triangle with the vertices (0,1), (1,1), (1,2)	**04**
	(c)	Find the volume of the solid generated by rotating the plane region bounded by $y = \frac{1}{x}$, x=1 and x=3 about the X axis.	**07**
Q.4	**(a)**	Evaluate $\int_0^{\pi} \int_0^{\sin\theta} r\, dr\, d\theta$	**03**
	(b)	Express f(x) = $2x^3 + 3x^2 - 8x + 7$ in terms of (x-2)	**04**

(c) Using Gauss-Jordan method find A^{-1} for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ **07**

OR

Q.4 **(a)** Using Cayley-Hamilton Theorem find A^{-1} for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **03**

(b) Evaluate $\int_0^\infty \frac{dx}{x^2+1}$ **04**

(c) Test the convergence of the series **07**

$\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \frac{x^4}{7\cdot 8} + \cdots$

Q.5 **(a)** Evaluate $\int_0^1 \int_1^2 xy\, dy\, dx$ **03**

(b) Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ **04**

(c) If u = f (x-y, y-z, z-x) then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **07**

OR

Q.5 **(a)** Find the directional derivatives of f = $xy^2 + yz^2$ at the point (2,-1,1) ,in the direction of i+2j+2k. **03**

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ **04**

(c) Evaluate $\iiint xyz\, dx\, dy\, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = 4$ **07**

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