(Time: 2½ hours)

Total Marks: 75

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt *any three* of the following: **15**

a. Reduce the matrix to normal form and find its rank where

$$A=\begin{bmatrix}1 & -1 & 3 & 6\\ 1 & 3 & -3 & -4\\ 5 & 3 & 3 & 11\end{bmatrix}$$

b. Examine for consistency the system of equations
$x-y-z=2;\quad x+2y+z=2;\quad 4x-7y-5z=2$ and solve them if found consistence.

c. Verify Cayley – Hamilton Theorem for the matrix A.

$$A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$$

d. Express in Polar from $-1+\sqrt{3}\,i$

e. Simplify $\dfrac{(\cos\theta-\sin\theta)^6(\cos5\theta-i\sin5\theta)^{-2}}{(\cos8\theta+i\sin8\theta)^{1/2}}$ using De-Moivre's theorem.

f. Prove that : $\therefore \sinh^{-1}x=\log\left(x+\sqrt{x^2+1}\right)$

2. Attempt *any three* of the following: **15**

a. Solve $y^2-x^2\dfrac{dy}{dx}=xy\dfrac{dy}{dx}$

b. Solve $\dfrac{dy}{dx}+2y\tan x=\sin x$

c. Solve $(p-2x)(p-y)=0$

d. Solve : $y=xp+\dfrac{1}{p}$

e. Solve : $(D^2+6D+9)y=5^x-\log 2$

f. Solve : $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-3y=0$

[TURN OVER]

3. Attempt *any three* of the following: **15**

a. Find the Laplace transform of $f(t)=\begin{cases}\cos t & 0<t<\pi \\ \sin t & t>\pi\end{cases}$

b. Evaluate by using Laplace transform $\int_0^{\infty} t^2 e^{-t}\sin t\, dt$

c. Find the Laplace transform of the following.

$$\frac{dy}{dt}+3y(t)+2\int_0^t y(t)dt=t; \quad given\ y(0)=0$$

d. Find the inverse Laplace transform of $\frac{s}{(s-2)^4}$

e. Find inverse Laplace transform of $\cot^{-1}(s)$

f. Find the Laplace transform of : $f(t)=\begin{cases}1 & 0<t<a \\ -1 & a<t<2a\end{cases}$ and $f(t)=f(t+2a)$

4. Attempt *any three* of the following: **15**

a. Evaluate : $\int_0^1\int_0^y xy\, e^{-x^2} dx\, dy$

b. Take Expression as a single integral and evaluate

$$\int_0^{a/\sqrt{2}}\int_0^{x} x\ dx\ dy+\int_{a/\sqrt{2}}^{a}\int_0^{\sqrt{a^2-x^2}} x\ dx\ dy$$

c. Evaluate $\int_0^{a}\int_0^{\sqrt{a^2-y^2}}\left(\sqrt{a^2-x^2-y^2}\right)dx\ dy$

d. Evaluate : $\iiint_V \frac{dx\ dy\ dz}{(x+y+z+1)^3}$ where V is the volume bounded by the planes, $x=0.\ y=0,\ z=0, and\ x+y+z=1.$

e. Evaluate $\iint xy(x+y)dx\,dy$ over the area between curve $y=x^2$ and the line $y=x$

f. Prove that the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{e^2}=1$ is $\frac{4\pi}{3}abc$

[TURN OVER]

5. **Attempt _any three_ of the following:** **15**

a. Evaluate $\int_0^{\infty} x^2 . e^{-h^2 x^2} . dx$

b. Evaluate $\int_0^{\pi} x \sin^6 x \; dx$

c. Show that : $\int_0^{\pi/2} \frac{\log\left(1 + a \sin^2 x\right)}{\sin^2 x} . dx = \pi\left[\sqrt{1+a} - 1\right]$

d. Show that : $\int_0^{\infty} \frac{\sin x}{x} . dx = \frac{\pi}{2}$

e. Find : $\frac{d}{dx}\left[erf(x) + erf_c(ax)\right]$

f. If $\phi(\alpha) = \int_{f(\alpha)}^{g(\alpha)} F(x, \alpha)\, dx$, write the rule to find $\frac{d\phi}{d\alpha}$ and hence prove that,

$$\frac{d}{dx}\left[erf\sqrt{x}\right] = \frac{e^{-x}}{\sqrt{\pi x}}$$
