

Q. P. Code: 20945

(Time: 2½ hours)

Total Marks: 75

- N. B.: (1) **All** questions are **compulsory**.
 (2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
 (3) Answers to the **same question** must be **written together**.
 (4) Numbers to the **right** indicate **marks**.
 (5) Draw **neat labeled diagrams** wherever **necessary**.
 (6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt any three of the following:

15

- a. Find the Adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & -2 \\ 5 & 1 & 8 \end{bmatrix}$$

- b. Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

- c. Verify Caley-Hamilton theorem for the given matrix, also find inverse if exists.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- d. Expand $(1 + \cos x + i \sin x)^n$
 e. Evaluate $(1 + i\sqrt{3})^{16} / (\sqrt{3} - i)^{17}$
 f. Express $\sec(x + iy)$ in $a + ib$ form

2. Attempt any three of the following:

15

- a. Solve the Differential Equation $(x - 4xy - 2y^2) dx + (y^2 - 4xy - 2x^2) dy = 0$
 b. Solve the Differential Equation $dy/dx + x^2y = x^5$
 c. Solve the following Equation $x^2p^2 - 2xpy + (2y^2 - x^2) = 0$
 d. Solve the following Equation $p(p + y) = x(x + y)$
 e. Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$
 f. Find the General Solution of the equation $(D^2 + 4)y = \sin 3x + e^x + x^2$

3. Attempt any three of the following:

15

- a. Evaluate $\int_0^\infty e^{-2t} \sin^2 t dt$
 b. Find the inverse Laplace transform for the function

$$F(s) = \frac{21-s^2}{s(s^2+4s+13)}$$

- c. Find Laplace transformation of the function

$$f(t) = te^{2t} \cos 3t$$

[TURN OVER]

Q. P. Code: 20945

- d. Obtain the Inverse Laplace transform of each of the given function

$$\frac{(s+1)}{s^3(s-3)^2}$$

- e. Find Inverse Laplace Transformation by convolution theorem for

$$F(s) = \frac{s}{(s^2+1)(s^2+4)}$$

- f. By using fundamental definition, find laplace transform of f(t)

$$F(t) = t, \quad 0 < t < 4 \\ = 5, \quad t > 4$$

4. Attempt any three of the following:

15

a.

Evaluate $\int_0^1 \int_0^2 e^{x+y} dx dy$

b.

Evaluate $\int_0^3 \int_0^{\sqrt{4-y}} \frac{dx dy}{(1+x^2+y^2)}$

c.

Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dx dy dz$

d.

Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx dy dz$

e.

Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xy dx dy$

f.

Solve $\iint r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

5. Attempt any three of the following:

15

a.

Evaluate $\int_0^{\pi/2} \sin^6 x \cos^7 x dx$

b.

Evaluate i) $\operatorname{erfc}(-x) + \operatorname{erfc}(x)$
ii) $\operatorname{erfc}(x) + \operatorname{erf}(x)$

c.

Evaluate $\int_0^{2a} x(2ax - x^2)^{1/2} dx$

[TURN OVER]

Q. P. Code: 20945

d. Evaluate $\int_0^{\pi/2} \sin^5 2x dx$

e. Evaluate $\int_0^1 \frac{x^7}{(1-x^4)^{1/2}} dx$

f. Evaluate $\int_0^1 \frac{(x^a - x^b)}{\log x} dx$
