Q. P. Code: 34331

(2 ½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt All(Each of 5Marks) **(15M)**

(a) Multiple Choice Questions.

i) Which of the following commands will create a list?
a) list l = list() b) list l = [] c) list l = ([1, 2, 3]) d) All of these

ii) The dot product of (1, 2, 3) and (1, −1, 0) is
a) 0 b) 2 c) 1 d) −1

iii) The dot product of (1, 2, 3) and (−1, 1, 0) is
a) 1 b) −1 c) 0 d) 2

iv) A linear equation with right hand side is equal to zero is called
a) A linear System b) Saturated
c) Homogeneous d) Non homogeneous

v) A vector whose norm is 1 is called___________vector
a) Null b) Besis c) Unit d) none of these

(b) Fill in the blanks for the following questions

i) Two vectors are said to be orthogonal if angle between them is ____
ii) The output when we execute list(“Hello”) is_____________.
iii) Set of all linear combinations of vectors is called _________
iv) If all the elements of a matrix have zero value is called as ________ matrix.
v) To add a new element to a list we use___________command.

(c) Answer the following questions

i) If u =(1, 2, −1) and v = (3, 2, −1) find norm u and norm v.
ii) Define the term Inner Product Space
iii) Solve (1•1) + (1•0) + (1•1)
iv) Define the term Characteristic equation
v) Find dot product of (1, 5), (4, −2)

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Q. 2 Attempt the following (Any THREE) (15M)

(a) Find the square root of complex number 8 – 6i

(b) Determine whether $v_1=(2, 2, 2)$, $v_2=(0, 0, 3)$ and $v_3=(0, 1, 1)$ span vector space R^3.

(c) Write a Python program to find conjugate of a complex number.

(d) Are the following vectors are linearly dependent
$v_1=(3, 2, 7)$, $v_2=(2, 4, 1)$ and $v_3=(1, -2, 6)$

(e) Express in polar and exponential form $1 + i\sqrt{3}$

(f) Check whether the set of all pairs of real numbers of the form (1, x) with operation (1, y) + (1, y′) = (1, y + y′) and k (1, y) = (1, ky) is a vector space.

Q. 3 Attempt the following (Any THREE) (15M)

(a) Find the angle between the two vectors $a = (2,3,4)$ and $b=(1, -4,3)$ in IR^3.

(b) Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 4 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \quad c = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \quad D = [2 \;\; 4 \;\; 3 \;\; 1]$$

Compute the following if they exists.
a) A +B b) 3A c) B + 2D

(c) Write a python program to enter a matrix and check if it is invertible. if invertible exists then find inverse.

(d) Check whether the set of functions are Linearly independent?
$2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$.

(e) Consider Subspace $U_1\{(x, y, w, z) : x - y = 0\}$ and $U_2\{(x, y, w, z) : x = w, y = z\}$ Find a basis and dimension of
i) U_1 ii) U_2 iii) $U_1 \cap U_2$.

(f) If V and W are two subsets of a vector space V such that U is a subset of W then show that W^0 is a subset of U^0 where U^0, W^0 are annihilator of U and W respectively.

Q. 4 Attempt the following (Any THREE) (15)

(a) Solve the following system by Gaussian elimination method.

$$\begin{aligned} y - z &= 3 \\ -2x + 4y - z &= 1 \\ -2x + 5y - 4z &= -2 \end{aligned}$$

(b) Find the orthonormal basis for subspace IR^4 whose generators are
$v_1 = (1, 1, 1, 1)$, $v_2 = (1, 2, 4, 5)$, $v_3 = (1, -3, -4, -2)$
Using Gram Schmidt orthogonali sation Method.

(c) Let $a = (3,0)$, $b = (2,1)$ find vector in span $\{a\}$ that is closet to b is $b^{||a}$ and distance $||b^{\perp a}||$.

(d) Verify Pythagorean Theorem for u = (1, 0, 2, –4) and v = (0, 3, 4, 2)

(e) Find inner product, angle, orthogonality for
$P = -5 + 2x - x^2$, $q = 2 + 3x^2$

(f) Write a python program to find orthogonal projection u on v.

Q. 5 Attempt the following (Any THREE) **(15)**

(a) Find eigen Values and eigen vectors of

$$A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

(b) Express the following as a linear combination of $v_1=(-2, 1, 3)$, $v_2=(3, 1, -1)$ and $v_3=(-1, -2, 1)$ with $w= (6, -2, 5)$

(c) Let $T : |R^3 \rightarrow |R^2$ be a linear map defined by f (x,y,z) = (x+2y-z, x+y-2z)
Verify Rank T + Nullity T = 3.

(d) Let S be a subset of vector space V. Prove that $S^\perp$ is a subspace of V.

(e) Fill the table.

Vector space	Basis	Dimension
{0}		
IR^2	{(1,0),(0,1)}	
$P_2(x)$		3
$M_2(IR)$		4
IR	{1}	

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