Q. P. Code: 34332

(2½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt the following questions **(15M)**

(a) Choose the best choice for the following questions **(5M)**

i) The absolute value of 3 + 4i is:
a) 4 b) 5 c) 6 d) zero

ii) In GF(2) field, 1 + 1 is equal to
a) 1 b) 0 c) both a) and b) d) none of these

iii) How to declare the complex number in Python?
a) (3, 4) b) Complex(3, 4) c) Complex (3, 4i) d) None of these

iv) If a matrix is R × C and a vector is a C vector then the product is called
a) Matrix-Matrix b) Vector-Matrix
c) Vector-Vector d) Matrix-Vector

v) Suppose t =(1, 2, 4, 3), which of the following is incorrect?
a) print(t[3]) b) t[3] = 45
c) print(max(t)) d) print(len(t))

(b) Fill in the blanks for the following questions **(5M)**

i) Any complex number multiplying by i, rotate it by ____________
ii) Set of all linear combinations of vectors is called ___________
iii) A rectangular array of m rows and n columns is called a ____________
iv) Norm of Vector (1, 2, 3) is ____________
v) Every Subset of a linearly independent set is linearly _____________

(c) Answer the following questions **(5M)**

i) Solve: 1•1 + 0•1 + 0•0 + 1•1
ii) Find dot product of (1, 2), (3, 4)
iii) Show with example matrix representation in python
iv) Define the term Basis
v) Define the term Inner Product Space

Q. 2 Attempt the following (Any THREE)(Each of 5Marks) **(15M)**

(a) Find the square root of complex number –5 + 12i
(b) Show that vectors $v_1=(1, 0, 1)$, $v_2=(2, 1, 4)$ and $v_3=(1, 1, 3)$ do not span vector space.
(c) Write a Python program to rotate a complex no by 90^0, 180^0 and 270^0

(d) Check whether the vectors are linearly dependent
$v_1=(1, -2, 1)$, $v_2=(2, 1, -2)$ and $v_3=(7, -4, 1)$.

(e) Express $[(3 + 2i)/(2 + i)(1 - 3i)]$ in the form $x + iy$

(f) Check whether the set of all pairs of real numbers of the form $(1, x)$ with operation $(1, y) + (1, y') = (1, y + y')$ and $k (1, y) = (1, ky)$ is a vector space.

Q. 3 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 4 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \quad c = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \quad D = [2 \;\; 4 \;\; 3 \;\; 1]$$

Compute the following if they exists.
a) $A + B$ b) $3A$ c) $B + 2D$

(b) Find the dimension of the vector space spanned by the vectors $(1, 1, -2, 0, -1)$, $(1, 2, 0, -4, 1)$, $(0, 1, 3, -3, 2)$, $(2, 3, 0, -2, 0)$ and also find the basis.

(c) Check whether the set of functions are Linearly independent?
$2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$.

(d) Explain Matrix-Vector and Vector-Matrix multiplication with example.

(e) Write a python program to enter a matrix and check if it is invertible. if invertible exists then find inverse.

(f) Show that vector $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ of R^3 form a basis of R^3

Q. 4 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) If $u = (2, 3, -1)$ and $v = (6, -3, -2)$
Find a) $d(u, v)$ b) $u - v$ c) $2u + 3v$

(b) Verify Pythagorean Theorem for $u = (1, 0, 2, -4)$ and $v = (0, 3, 4, 2)$

(c) If $x, y, z \geq 0$
Show that $(x^2 + y^2 + z^2)^{1/2} \geq (1/13)(3x + 4y + 12z)$

(d) Find inner product, angle, orthogonality for
$P = -5 + 2x - x^2$, $q = 2 + 3x^2$

(e) Find the vector orthogonal to both $u = (-6, 4, 2)$ and $v = (3, 1, 5)$

(f) Write a python program to find orthogonal projection u on v.

Q. 5 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Express the following as a linear combination of $v_1=(-2, 1, 3)$, $v_2=(3, 1, -1)$ and $v_3=(-1, -2, 1)$ with $w= (6, -2, 5)$

(b) Write a python program to convert a 2×2 matrix to row echelon form

(c) Verify Cauchy's Schwartz's inequality $u = (1, 2, -1)$ and $v = (3, 2, -1)$

(d) Find eigen Values and eigen vectors of

$$A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

(e) Construct an orthonormal basis of R^2 by Gram Schmitt Process
$S = \{(3, 1), (4, 2)\}$