

( 2 ½ Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.  
 2) Figures to the right indicate marks.  
 3) Illustrations, in-depth answers and diagrams will be appreciated.  
 4) Mixing of sub-questions is not allowed.

Q.1 Attempt All (Each of 5 Marks)

(15M)

(a) Select correct answer from the following:

- A set with zero characters as a string is called \_\_\_\_\_ string.  
 (a) Null (b) Unit (c) binary (d) ternary
- A set of minimum edges required to delete for disconnected graph.  
 (a) Flow (b) Cut set (c) bridges (d) none
- Chromatic number of a complete graph with  $n$  vertices is \_\_\_\_\_.  
 (a)  $n!$  (b)  $n$  (c)  $n+1$  (d)  $n-1$
- A network is a \_\_\_\_\_ graph.  
 (a) discrete (b) Regular (c) Connected (d) Multigraph
- The amount of material flowing into a vertex  $v$  must \_\_\_\_\_ to the amount flows out of the vertex.  
 a) equal (b) Less than (c) Greater than (d) None

(b) Fill in the blanks:

(Degree, string,  $8!$ ,  $n!$ ,  $n-1$ ,  $n$ ,  $n+1$ , saturated, Flow, 9, Parallel)

- The number of permutations of the letters in the word COMPUTER is \_\_\_\_\_.
- The value of  $C(7, 2)$  is \_\_\_\_\_.
- If  $N = (V, E)$  is a transport network, a function  $f$  from  $E$  to the nonnegative integers is called a \_\_\_\_\_.
- If two or more edges have same terminal vertices then these edges are called \_\_\_\_\_ edges.
- Ramsey number,  $R(3, 4) =$  \_\_\_\_\_.

(c) Define the following.

- Binomial Theorem
- Addition rule in counting problems
- Planar graph
- Clique
- Saturated edge

Q.2 Attempt the following (Any THREE)

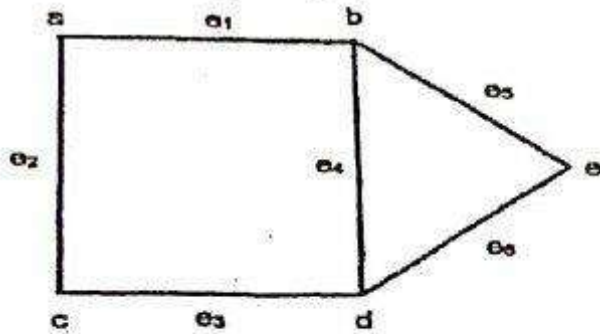
(15M)

- A farmer buys 3 cows, 2 goats and 4 hens from a man who has 4 cows, 3 goats and 8 hens. How many choices does the farmer have?
- Determine the coefficient on  $xyz^5$  in the expansion of  $(x + y + z)^7$
- Prove that the sum of first  $n$  natural numbers is  $\frac{n(n+1)}{2}$ .
- Determine all integer solutions to the equation  $x_1 + x_2 + x_3 + x_4 = 7$ , where  $x_i \geq 0$  for all  $1 \leq i \leq 4$

- (e) What is Sudoku puzzle? Explain the easiest way to solve Sudoku puzzle and write its three benefits.
- (f) For each  $n > 0$ , prove that 
$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$$

Q.3 Attempt the following (Any THREE) (15M)

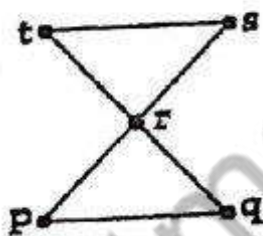
- (a) Explain colouring of graph and Chromatic numbers? Find the chromatic number of the given graph



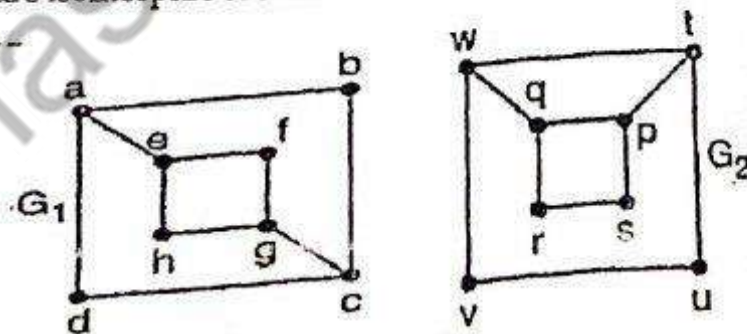
- (b) Define a regular graph. Draw a regular graph with five vertices
- (c) State and prove Ramsey's theorem.
- (d) Define adjacency matrix representation of a graph also draw the graph for

the given adjacency matrix 
$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- (e) Define Euler's path, Euler's circuit and Euler's graph also find an Eulerian circuit in the given graph.



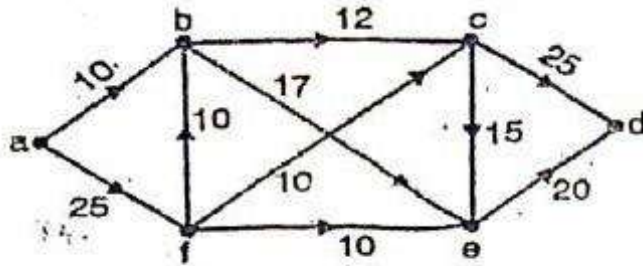
- (f) Explain isomorphism of graphs, Check whether the two graphs  $G_1$  and  $G_2$  are isomorphic or not.



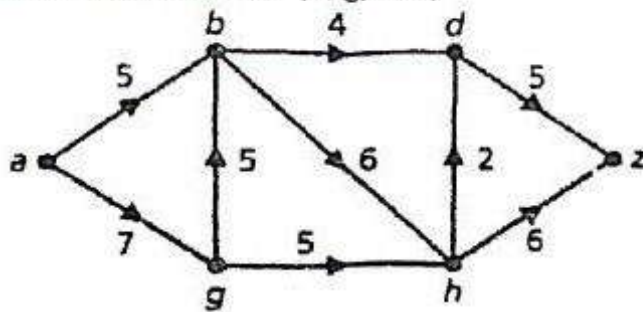
Q.4 Attempt the following (Any THREE)

(15)

- (a) Use Ford- Fulkerson algorithm to find the maximum flow for the following network:



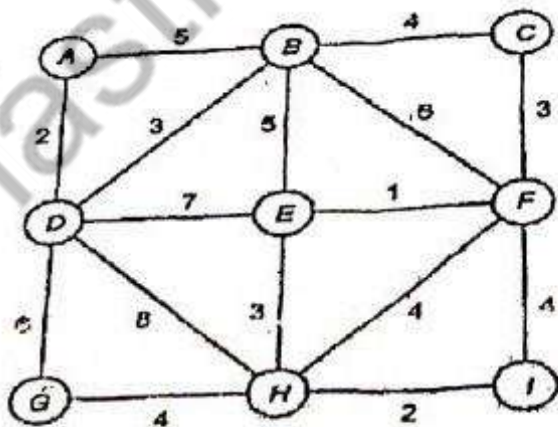
- (b) Define the capacity of cuts. Find the capacity of the cut ( P,Q), where  $P=\{a, b\}$  and  $Q=\{d, g, h, z\}$



- (c) What is a complete matching? Explain with an example.
- (d) Write permutations shown below in cycle notation, compute  $\pi_1\pi_2$  (product of two permutations) and inverse of  $\pi_1$ .
- $$\pi_1 = \begin{pmatrix} a & b & c & d & e & f \\ e & f & d & a & b & c \end{pmatrix}, \pi_2 = \begin{pmatrix} a & b & c & d & e & f \\ f & c & a & e & d & b \end{pmatrix}$$
- (e) Suppose we are colouring the vertices of the square using black and white colour. Draw the colouring fixed by all possible transformations and explain it in brief.
- (f) What is the integer solution of linear programming problems?

Q.5 Attempt the following (Any THREE) (15)

- (a) From a standard deck 52 cards, In how many ways we can draw three cards in succession without replacement?
- (b) Find minimum spanning tree of following graph using Kruskal's algorithm.



- (c) Explain Polya's enumeration formula.

- (d) Explain Burnside's Lemma
- (e) Expand  $(x + y)^7$  using binomial theorem and find the coefficient of  $x^6y$ .

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