

**Q. P. Code: 19834**

(2 1/2 Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.  
 2) Figures to the right indicate marks.  
 3) Illustrations, in-depth answers and diagrams will be appreciated.  
 4) Mixing of sub-questions is not allowed.

**Q. 1 Attempt All (Each of 5Marks)**

(15M)

## (a) Select correct answer from the following:

1. The product of two consecutive natural number is always divisible by \_\_\_\_\_.  
 (a) 0 (b) 2 (c) 6 (d) 3
2. Pascal triangle is used to find the coefficients of \_\_\_\_\_ expansion  
 (a) Multinomial (b) binomial (c) exponential (d) all of these
3. A vertex with degree zero is called \_\_\_\_\_.  
 (a) Pendant (b) isolated (c) incident (d) none of the above
4. The walk in which no vertex is repeated more than one is called \_\_\_\_\_.  
 (a) cycle (b) trail (c) path (d) cut
5. An edge  $(x, y)$  with flow  $\phi(x, y)$  and capacity  $c(x, y)$  is said to be saturated when  $\phi(x, y) = c(x, y)$ .  
 (a)  $\neq$  (b)  $\leq$  (c)  $\geq$  (d) =

## (b) Fill in the blanks.

(pseudograph, zero, 2, combination, least)

1. \_\_\_\_\_ is the selection of  $r$  objects from a set of  $n$  objects.
2. Every non empty set of positive integers has a \_\_\_\_\_ element.
3. A graph with parallel edges and loop is called a \_\_\_\_\_.
4. Let  $G$  be a connected planar graph with  $n$  vertices,  $m$  edges and  $f$  faces then  $n-m+f=$  \_\_\_\_\_.
5. Sink 'z' in network has \_\_\_\_\_ outdegree.

## (c) Define the following.

1. Binomial Theorem
2. Regular graph
3. Clique
4. Labelling of source
5. Augmenting path

**Q. 2 Attempt the following (Any THREE)**

(15M)

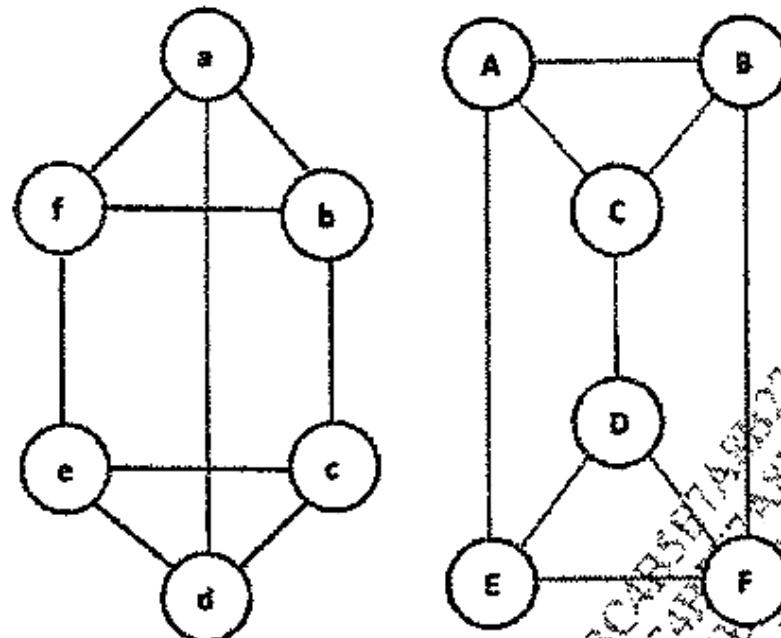
- (a) A student taking a history examination is directed to answer any seven of 10 essay question. In how many ways the student can answer the examination if there is no concern about the order?  
 In how many ways the student must answer three questions from the first five and four questions from last five?
- (b) Determine the coefficient of  $xyz^2$  in the expansion of  $(x + y + z)^4$ .
- (c) Show that for all positive integers  $n$ ,  $7^n - 2^n$  is divisible by 5.

- (d) How many integer-valued solutions are there for the equation  
 $x_1 + x_2 + x_3 + x_4 + x_5 = 132$ , all  $x_i \geq 0$
- (e) Explain the application of Combinatorics in graph theory.
- (f) For each  $n > 0$ , prove that  
 $\binom{n}{0} 2^0 + \binom{n}{1} 2^1 + \binom{n}{2} 2^2 + \dots + \binom{n}{n} 2^n = 3^n$

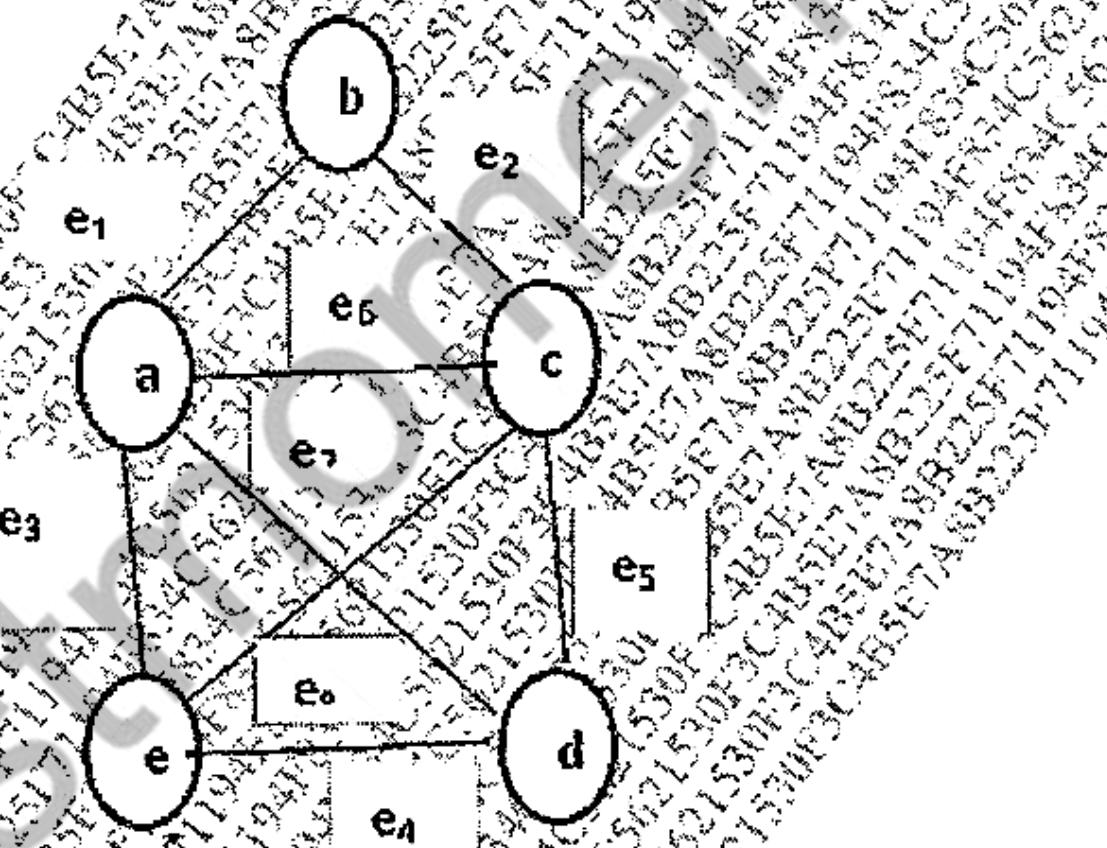
**Q. 3 Attempt the following (Any THREE)**

(15M)

- (a) Show that following graphs are isomorphic.

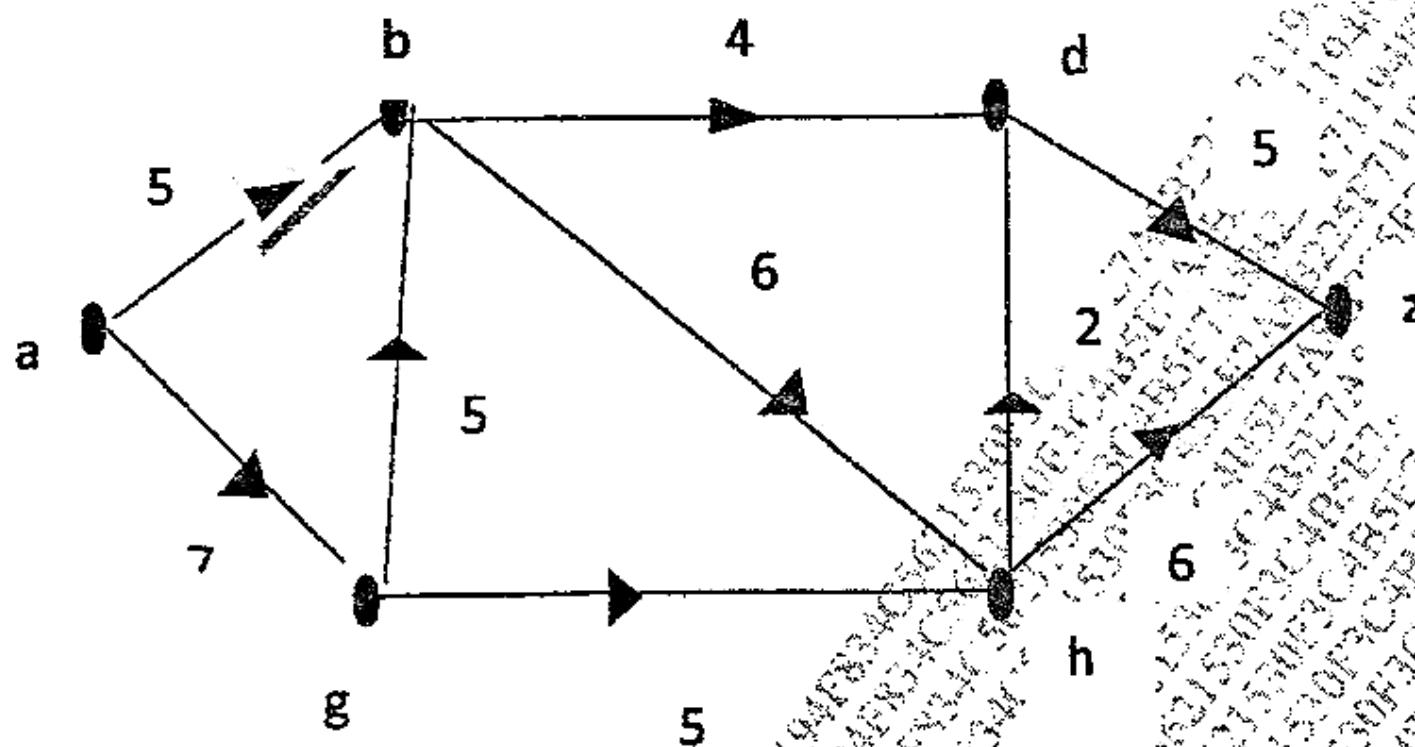


- (b) Draw a tree whose prüfer ( $T$ ) = 6643143 with vertex set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- (c) Define Planar graph, also represent  $K_4$  in a Planar representation.
- (d) Explain Eulerian and Hamiltonian graph with the help of examples.
- (e) State pigeonhole principle and Show that if any five numbers from the set  $\{1, 2, 3, 4, 5, 6, 7, 8\}$  are chosen, then two of them will add up to 9.
- (f) Define incident matrix and also find the incident matrix of the given graph.

**Q. 4 Attempt the following (Any THREE)**

(15)

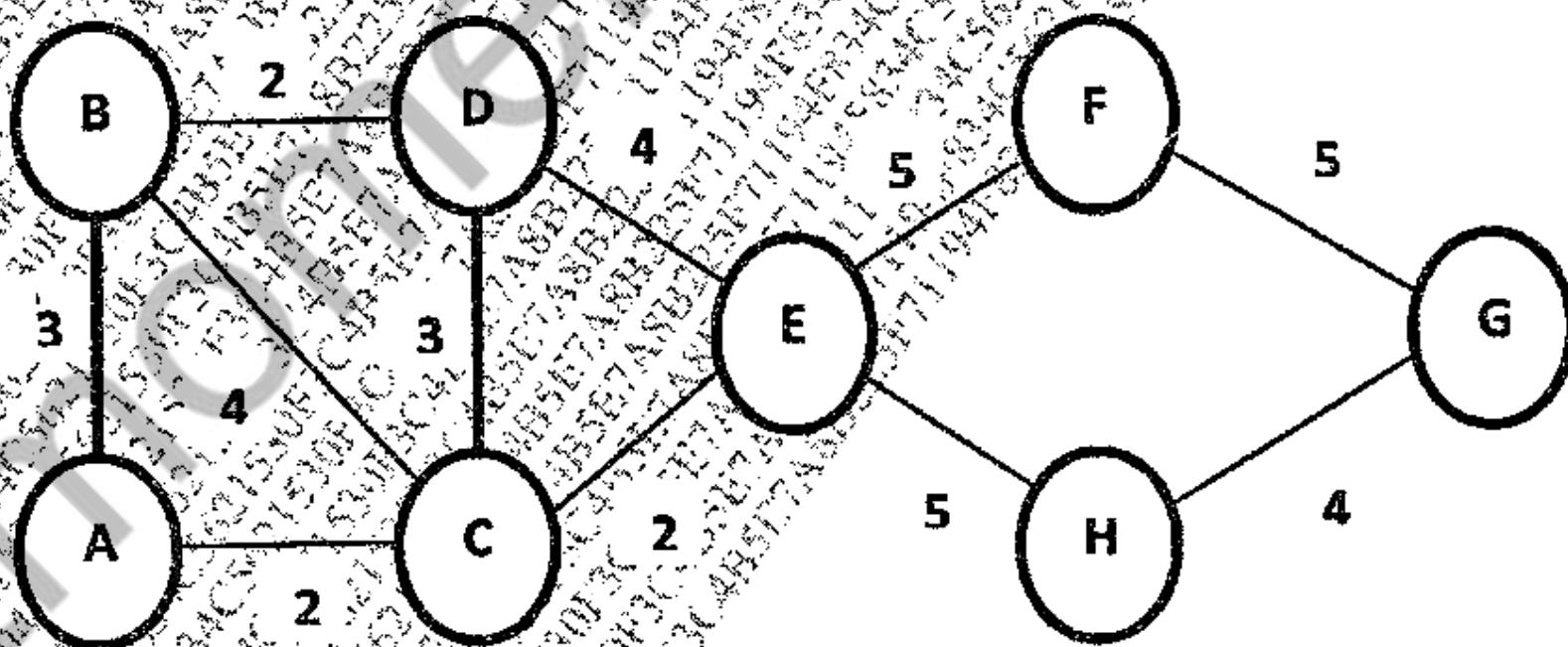
- (a) Explain the relation between flows and cuts.
- (b) Find maximum flow of the following network.



- (c) Explain Ford-Fulkerson's labelling algorithm?  
 (d) In how many distinct ways can we colour the vertices with black and white of a square? Write all the transformations where colours are fixed.  
 (e) Explain Cycle index.  
 (f) What is matching in Bipartite graph?

**Q. 5** Attempt the following (Any THREE) (15)

- (a) In how many ways can we arrange the letters in TALLAHASSEE? How many of these arrangements have no adjacent A's?  
 (b) Find minimum spanning tree of the following graph using prim's algorithm



- (c) Define Chromatic number with example.  
 (d) What are the integer solutions of linear programming?  
 (e) State first principle and second principle of mathematical induction.

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