**Q.P. Code: 33412**

**(2½ Hours)**

**[Total Marks: 75]**

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labelled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

**1. Attempt *any three* of the following:** **15**

a. Water accounts for roughly 60% of total body weight. Assuming it can be categorized into six regions, the percentages go as follows. Plasma claims 4.5% of the body weight and is 7.5% of the total body water. Dense connective tissue and cartilage occupies 4.5% of the total body weight and 7.5% of the total body water. Interstitial lymph is 12% of the body weight, which is 20% of the total body water. Inaccessible bone water is roughly 7.5% of the total body water and 4.5% total body weight. If intracellular water is 33% of the total body weight and transcellular water is 2.5% of the total body water, what percent of total body weight must the transcellular water be and what percent of total body water must the intracellular water be?

b. What is a mathematical model? With the help of a flowchart, explain the of solving an engineering problem.

c. Discuss the aspects of round-off errors while storing floating point numbers in computer.

d. Use zero- through fourth-order Taylor series expansions to approximate the function:
$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$
from $x_i = 0$ with $h = 1$. That is, predict the function's value at $x_{i+1} = 1$.

e. Explain Total numerical error, formulation error and data uncertainty.

f. Define accuracy and precision. What are round-off errors? Explain.

**2. Attempt *any three* of the following:** **15**

a. Determine the real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 - 91x^2 + x^5$ between 0.5 and 1.0 correct up to 3 decimal places using bisection method.

b. Determine the positive real root of $\ln(x^4) = 0.7$ between 0.5 and 2 using method of false position.

c. Solve: $x - 0.8 - 0.2\sin x = 0$ using Newton Raphson method correct upto 4 decimal places starting with initial value 0.

d. From the table of Bessel function $J_n(1)$, estimate the value of $J_{\frac{3}{2}}(1)$

| $n$ | $-1$ | $-\frac{3}{4}$ | $-\frac{1}{2}$ | $-\frac{1}{4}$ | 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
|---|---|---|---|---|---|---|---|---|---|
| $J_n(1)$ | –0.4401 | 0.0447 | 0.4311 | 0.6694 | 0.7652 | 0.7522 | 0.6714 | 0.5587 | 0.4401 |

e. Find *f (8.4)* if *f (8.1)* = 16.94410, *f (8.3)* = 17.56492, *f (8.6)* = 18.50515, *f (8.7)* = 18.82091 using Lagrange's Interpolation formula.

f. Using the necessary interpolation formula find *f(1)* and *f(1.5)* from the table:

| *x* | –1 | 0 | 2 | 3 |
|---|---|---|---|---|
| *f(x)* | –8 | 3 | 1 | 12 |

**[TURN OVER]**

**Q.P. Code: 33412**

**3. Attempt *any three* of the following:** **15**

a. Solve the following system by using the Gauss-Jordan elimination method.

$$x + y + z = 5$$
$$2x + 3y + 5z = 8$$
$$4x + 5z = 2$$

b. Use the Gauss-Seidel iterative technique to find approximate solutions to

$$a + b + 2c = 1$$
$$2a - b + d = -2$$
$$a - b - c - 2d = 4$$
$$2a - b + 2c - d = 0$$

c. Given log 280 = 2.4472, log 281 = 2.4487, log 283 = 2.4518, log 286 = 2.4564. Find $\left[\frac{d}{dx}(\log x)\right]_{x=280}$

d. Evaluate the following using Simpson's 3/8th rule.

$$\int_0^{\pi} \frac{\sin^2\theta}{5 + 4\cos\theta}\, d\theta$$

e. Use Euler's method to approximate the solution for

$$y' = t^{-2}(\sin 2t - 2ty), \quad 1 \le t \le 2, y(1) = 2 \text{ with } h = 0.5$$

f. Solve $y' = y - t^2 + 1, y(0) = 0.5, 0 \le t \le 2$ using Runge Kutta 4th order method with $h$ = 0.5

**4. Attempt *any three* of the following:** **15**

a. Fit a second order polynomial to the data given below:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 2.1 | 7.7 | 13.6 | 27.2 | 40.9 | 61.1 |

b. Fit a straight line to the given data regarding $x$ as the independent variable.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 1200 | 900 | 600 | 200 | 110 | 50 |

c. Consider the data below:

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 1 | 7 | 11 | 21 |

Use linear least-squares regression to determine a function of the form $y = be^{mx}$ for the given data by specifying b and m.

d. A farmer can plant up to 8 acres of land with wheat and barley. He can earn ₹ 5,000 for every acre he plants with wheat and ₹ 3,000 for every acre he plants with barley. His use of a necessary pesticide is limited by federal regulations to 10 gallons for his entire 8 acres. Wheat requires 2 gallons of pesticide for every acre planted and barley requires just 1 gallon per acre. What is the maximum profit he can make? Solve graphically.

e. The Bead Store sells material for customers to make their own jewelry. Customer can select beads from various bins. Grace wants to design her own Halloween necklace from orange and black beads. She wants to make a necklace that is at least 12 inches long, but no more than 24 inches long. Grace also wants her necklace to contain black beads that are at least twice the length of orange beads. Finally, she wants her necklace to have at least 5 inches of black beads.
Find the constraints, sketch the problem and find the vertices (intersection points).

**[TURN OVER]**

f. A garden shop wishes to prepare a supply of special fertilizer at a minimal cost by mixing two fertilizers, A and B.
The mixture is to contain: at least 45 units of phosphate, at least 36 units of nitrate at least 40 units of ammonium. Fertilizer A costs the shop \$.97 per pound. Fertilizer B costs the shop \$1.89 per pound. Fertilizer A contains 5 units of phosphate and 2 units of nitrate and 2 units of ammonium, fertilizer B contains 3 units of phosphate and 3 units of nitrate and 5 units of ammonium. How many pounds of each fertilizer should the shop use in order to minimize their cost?

**5. Attempt *__any three__* of the following:** **15**

a. The amount of bread (in hundreds of pounds) X that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function $f(x)$, given by

$$f(x) = A \cdot x \quad \text{for } 0 \le x \le 5$$
$$= A(10 - x) \quad \text{for } 5 \le x \le 10$$
$$= 0, \text{ otherwise}$$

i. Find the value of A such thatf(x) is a probability density function.
ii. What is the probability that the number of pounds of bread that will be sold tomorrow is
- more than 500 pounds
- less than 500 pounds
- between 250 and 750 pounds?

b. Suppose the life in hours of a certain kind of radio tube has the probability density function:

$$f(x) = \frac{100}{x^2}, \text{ when } x \ge 100$$
$$= 0, \text{ when } x < 100$$

What is the probability that none of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation? What is the probability that all three of the original tubes will have been replaced during the first 150 hours?

c. The diameter of an electric cable; say X, is assumed to be a continuous random variable with p.d.f.

$$f(x) = 6x(1 - x), \qquad 0 \le x \le 1.$$

i. Check that the function is p.d.f.
ii. Determine a number $b$ such that $P(X < b) = P(X > b)$

d. If 20% of the bolts produced by a machine are defective, determine the probability that, out of 4 bolts chosen at random, (i) 1, (ii) 0, and (iii) at most 2 bolts will be defective.

e. A department in a works has 10 machines which may need adjustment from time to time during the day. Three of these machines are old, each having a probability of 1/11 of needing adjustment during the day, and 7 are new having corresponding probabilities of 1/21. Assuming that no machine needs adjustment twice on the same day, determine the probability that on a particular day
i. just 2 old and no new machines need adjustment.
ii. If just 2 machines need adjustment, they are of the same type.

f. In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.

---