(Time: $2\frac{1}{2}$ hours)

[Marks: 75]

Please check whether you have got the right question paper.

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculator is **allowed**.

1. Attempt *any three* of the following: **15**

a. Explain the concept behind the conservation laws in science and engineering with examples.

b. Explain the terms: i) Significant figures, ii) Accuracy, iii) Precision, iv) Truncation error, v) Round-off error.

c. Evaluate e^{-5} using the two formulae:

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

And

$$e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots}$$

And compare with the true value 6.737947 X 10^{-3}. Use five terms to evaluate each series and compute true and approximate relative errors as terms are added.

d. Use Taylor series expansions with n = 0 to 6 to approximate $f(x) = cosx$ at $x_{i+1} = \pi/3$ on the basis of the value of f (x) and its derivatives at $x_i = \pi/4$.

e. Evaluate and interpret the condition number for

$$f(x) = \frac{sinx}{+ cosx} \; for\; x = 1.0001\pi 1$$

f. What are total numerical errors? Discuss stability and condition of a mathematical problem.

2. Attempt *any three* of the following: **15**

a. Find the roots of the equation

$$2x - 3sinx - 5 = 0$$

using Regula-Falsi method correct up to 3 decimal places.

b. Find the roots of the equation

$$x^4 - 2.03x^3 + 2.03x^2 - 2.03x + 1.03 = 0$$

in the neighbourhood of x = 1 using Newton Raphson method correct upto 5 decimal places.

c. Determine the real root of $f(x) = 4x^3 - 6x^2 + 7x - 2.3$ using bisection method correct upto 3 decimal places.

[TURN OVER]

d. Given log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990 and log 7 = 0.8451. Using Lagrange's formula, find log 47.

e. The following table shows the number of students and range of marks. Find the number of students who have secured less than 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	45	32	27	15

f. Given:

X:	1	2	3	4	5	6	7	8
f(x)	0.01	0.004	0.02	0.12	0.15	0.257	0.325	0.231

Find f(7.5)using Newton's backward interpolation formula

3. Attempt *any three* of the following: **15**

a. Solve the following simultaneous equations by Gauss – Seidel method:

$10x_1 + x_2 + x_3 = 12$

$2x_1 + 10x_2 + x_3 = 13$

$2x_1 + 2x_2 + 10x_3 = 14$

b. Solve the following system of equations by Gauss-Jordan method

$8x + 2y - 3z = 4$

$2x - 5y - 6z = 8$

$7x - 2y + 5z = 15$

c. The table for $f(x)$ is given below. Evaluate $\frac{d}{dx} f(x)$ and $\frac{d^2}{dy_2} f(x)$ at x = 0.1

x	0.0	0.1	0.2	0.3	0.4
f(x)	1.000	0.9975	0.9900	0.9776	0.9604

d. Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5 + 4\cos x} dx$ using Simpson's 3/8th rule.

e. Solve $\frac{dy}{dx} = \log(x + y)$; $y(1) = 2$ for $x = 1.2$ and $x = 1.4$ using Euler's modified method, taking h = 0.2.

f. By using Runge-Kutta method of order 4 to evaluate $y(2.4)$ from the following differential equation

$y'(x) = f(x, y)$ where $f(x, y) = (x + 1)y$

Initial condition $y(2) = 1$, $h = 0.2$ correct upto 4 decimals.

[TURN OVER]

4. Attempt *<u>any three</u>* of the following: **15**

a Use least square technique to fit a line $y = a_0 + a_1 x$ a_0, a_1 constants for the following data

x	5	15	18	20	30	35
y	12	14	20	18	28	22

b Fit a second degree parabola for the following

x	-2.5	-2	-1.5	-0.5	0	0.5	1.5
y	14.32	14.83	15.27	15.47	16.26	16.79	17.23

c Use multiple regression to fit the following data:

x_1	x_2	y
0	0	5
2	1	10
2.5	2	9
1	3	0
4	6	3
7	2	27

d Solve the following LPP graphically Minimize $z = 2x_1 + x_2$ subject to $x_1 + 2x_2 \leq 40$; $3x_1 + x_2 \geq 30$; $4x_1 + 3x_2 \geq 60$; $x_1 x_2 \geq 0$

e In a college, scholarship is to be given to the students of XI and XII classes. It is decided that at least 5 students from class XI and at least 4 students from class XII should get scholarship. Each scholarship holder of class XI will get Rs. 30 per month and each scholarship holder of class XII will get Rs. 40 per month. The total number of scholarship holders should not be less than 10 but should not be greater than 15. How many students of each class should be selected so as to (i) minimise (ii) maximise the amount spent on scholarship?

f Solve LPP graphically.
A company produces two products x and y each of which requires three types of processing. The length of time for processing each unit and profit per unit are given below

	Product x	Product y	Capacity
Process I	12	12	840
Process II	3	6	300
Process III	8	4	480
Profit	5	7	

How many units of each product the company produce per day to maximize profit?

[TURN OVER]

5. **Attempt *any three* of the following:** **15**

a. State and explain the properties of distribution functions.

b. A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) Find k, (ii) Evaluate $P(X < 6), P(X \geq 6)$, and $P(0 < X < 5)$, (iii) If $P(X \leq c) > \frac{1}{2}$, find the minimum value of c, and (iv) Determine the distribution function of X.

c. Prove that the geometric mean G of the distribution $dF = 6(2-x)(x-1)\,dx, 1 \leq x \leq 2$
is given by 6 log (16G) = 19.

d. The time between release from prison and the commission of another crime is uniformly distributed between 0 and 5 years for a high-risk group. Give the equation and graph the pdf for X, the time between release and the commission of another crime for this group. What percent of this group will commit another crime within two years of their release from prison?

e. A manufacturer claims that at most 10 per cent of his product is defective. To test this claim, 18 units are inspected and his claim is accepted if among these 18 units, at most 2 are defective. Find the probability that the manufacturer's claim will be accepted if the actual probability that a unit is defective is
(i) 0.05 (ii) 0.10 (iii) 0.15 (iv) 0.20.

f. The lifetimes in years for a particular brand of cathode ray tube are exponentially distributed with a mean of 5 years. What percent of the tubes have lifetimes between 5 and 8 years? Draw a graph of the pdf and shade the area which represents the probability of the event 5 < X < 8, where X represents the lifetimes.