Numerical & Statistical methods

Q. P. Code : 33414

(2½ Hours) [Total Marks: 75

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt **any three** of the following: 15
a. Explain Conservation Laws and Engineering Problems.
b. Explain the following with examples
   i) Blunders
   ii) Formulation Errors
   iii) Data Uncertainty
   iv) Total Numerical Errors
c. Explain Floating Point representation and Errors in floating point arithmetic.
d. Use zero through third order Taylor series expansions to predict f(3) for
   $F(x) = 25x^3 - 6x^2 + 7x - 88$
   Using a base point at $x = 1$. Compute the true percent relative error for each approximation.
e. Evaluate $y = x^3 - 7x^2 + 8x - 0.35$ at 1.37 use 3 digit and 4 digit arithmetic and find the significant digits lost. Also find the relative error after rounding-off.
f. Evaluate f(1) using Taylors series for f(x), where, $f(x) = x^3 - 3x^2 + 5x - 10$

2. Attempt **any three** of the following: 15
a. Define and express each of the Δ, ∇, δ. and μ in terms of E.
b. Find the polynomial using Lagrange's interpolation polynomial which agrees with the table below given values. Hence obtain the value of f(x) at x = 2

x	0	1	3	4
f(x)	-12	0	6	12

c. Find the Newton's forward difference interpolation polynomial which agrees with the table below given values. Hence obtain the value of f(x) at x = 6

x	0	1	2	3	4	5
f(x)	-5	-10	-9	4	35	90

d. Obtain the root for each of the following equations using Regula Falsi Method by 5 iterations.
   $F(x) = x^3 - 8x + 40 = 0$ upto 4 decimal places with $x_0 = -5$ and $x_1 = -4$.
e. Obtain the root for each of the following equations by Newton Raphson Method by 5 iterations.
   $F(x) = 2x^3 + 5x^2 + 5x + 3 = 0$ up to 4 decimal places
f. Explain Bisection method. Find the approximate root of $x^3 - x - 4 = 0$ by Bisection method up to 4 decimal places. Perform 4 iterations.

3. Attempt **any three** of the following: 15
a. Solve the following system of equation by Gauss-Jordan elimination method.
   $5x - y + z = 10$; $2x + 4y = 12$; $x + y + 5z = -1$.
b. Solve the following system of equation, correct to four places of decimals by Gauss-Seidal method perform 4 iteration use pivoting if necessary:
   $30x - 2y + 3z = 75$; $2x + 2y + 18z = 30$; $x + 17y - 2z = 48$

[TURN OVER]

c. Evaluate $\int_0^2 \log(1+x)^{1/2} \, dx$ using Simpson's one third rule with 8 sub-intervals.

d. Evaluate $\int_0^{10} (x + 1/x) \, dx$ by Trapezoidal Rule with 10 sub-intervals and find the error.

e. Find the values of $y(0.1)$ and $y(0.2)$ using Euler's modified methods with $h = 0.1$ given that $dy/dx + y/x = y^2$, $y(1) = 1$.

f. Find $y(0.2)$ and $y(0.4)$ taking $h(0.2)$ by second order Runge-Kutta method given that $dy/dx = (y-x)/(y+x)$, $y(0) = 1$

4. Attempt **any three** of the following:

15

a. An electrical firm manufactures circuit boards in two configurations, say (1) and (2). Each circuit board in configuration (1) requires 1 component of A, 2 components of B and 2 components of C, each circuit board in configuration (2) requires 2 components of A, 2 components of B and 1 Component of C. Total components available are 30, 30 and 25 of A, B and C respectively. If the profit realized upon sale is Rs 200 per circuit board in configuration (1) and Rs 150 per circuit in configuration (2). How many circuit boards of each configuration should the firm manufacture so as to maximize profit? Formulate the problem as Linear programming model and solve graphically.

b. A diet is to contain at least 400 units of carbohydrates, 500 units of fat and 300 units of protein. Two foods are available F1 which costs Rs 2 per units and F2 which costs Rs 4 per unit. A unit of food F1 contain 10 units of carbohydrates, 20 units of fat and 15 units of protein and a unit of food F2 contains 25 units of carbohydrates, 10 units of fat and 20 units of protein. Find the minimum cost for a diet that consists of a mixture of these two foods and also meets the minimum nutrition requirements. Formulate the problem as Linear programming model.

c. Explain the Applications of Linear Programming in Business and Industry.

d. Find the straight line approximation to the following data.

X	71	68	73	69	67	65	66	67
Y	69	72	70	70	68	67	68	64

e. Find the least square polynomial approximation of degree two equation from data below:

X	1	2	3	4	5	6	7	8	9
Y	2	6	7	8	10	11	11	10	9

f. Obtain a regression plane by using multiple regression to fit the following data.

X	0	1	2	3	4
Z	1	2	3	4	5
Y	13	17	19	21	26

5. Attempt **any three** of the following:

a. Explain the following
   i) Random Variable
   ii) Probability density function
   iii) Probability mass function

15

b. The probability distribution function of a discrete random variable X is given by

X	-2	-1	0	1	2
P(X=x)	0.1	0.15	0.2	0.15	0.4

Find i) $P(X \leq 0)$ ii) $P(X \geq -1)$. Also obtain the probability distribution of $y = x^2$

[TURN OVER]

c. A random variable X has the following probability distribution.

X	1	2	3	4	5	6
P(X=x)	3C	5C	7C	9C	11C	13C

   i) Find C   ii) P(X>=2)   iii) P(0<X<4)

d. If $X \sim N (\mu = 30, \sigma = 7)$. Find
   i)   P(X<20)
   ii)   P (33<X<45)
   iii) P (15 < X < 25)

e. It is observed that 20% of the students in a class are vegetarians. If 4 students are selected at random from this class, what is the probability that
   i)   Exactly one student is vegetarian
   ii)   At least two of them are vegetarian

f. A senior citizen receives on an average 2.5 telephone calls during his afternoon nap period 1400-1405 hrs. Find the probability that on a certain day, he receives
   i) No telephone calls   ii) Exactly 4 calls during the same period. [Given $e^{-2.5} = 0.0821$]

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