Q.P. Code: 31173

(2 ½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt All (Each of 5Marks) **(15M)**

(a) Select correct answer from the following:

1) In which of the following method, we approximate the curve of solution by the tangent in each interval .
a) Simpson's Method
b) Euler's method
c) Newton's method
d) None of the above

2) $\int 1/(9x^2 + 25)\ dx =$
a) $(3/5)\tan^{-1}(3x/5) + c$
b) $(1/9)\tan^{-1}(3x/5) + c$
c) $(3/5)\tan^{-1}(5x/3) + c$
d) $(1/15)\tan^{-1}(3x/5) + c$

3) A function is said to be invertible if and only if it is______________
a) Bijective b) injective c) Inflexion d) Surjective

4) $\lim_{x\to\infty} 7/2x =$
a) 1 b) infinite c) zero d) None

5) If $f(x, y) = x^3y^3 + y^3 + 1$ then $f_x(x, y)$ is
a) $3x^2$ b) $3xy$ c) y^3x d) None

(b) Fill in the blanks:
(*continuous*, ∞, (4i+5j)/41, (4i+5j)/31, -∞, e^x, derivative, x – 3 log|x + 3| + c)

1. $\lim_{x\to\infty}(5 - 2x) =$_____________.
2. The derivative of e^x is__________________.
3. Unit vector of 4i+5j is______________.
4. $\int x/(x + 3)\ dx =$ _____________
5. The rate of change of one variable with respect to another is called _____________.

(c) Answer the following in one line

1. Define Tangent Plane
2. Define Critical Point
3. Define the term Definite Integral
4. Evaluate $\int_{\pi/3}^{2\pi} \sin x \; dx$
5. Linearization of a function

Q. 2 Attempt the following (Any THREE) **(15M)**

(a) Show that $\lim_{x\to 1} 2x^2 + 3x - 4 = 1$

(b) Discuss the continuity of the function $f(x)=\sqrt{4-x^2}$

(c) Show that the function $f(x) = x^3 - 9x^2 + 30x + 7$ is always increasing.

(d) Find the relative extrema of $f(x) = 4xy - x^4 - y^4$ using both first and second derivative test.

(e) Using Newton's method find the approximate root for the equation $f(x)=x-\cos x$

(f) Divide 100 into two parts such that sum of their square is minimum.

Q. 3 Attempt the following (Any THREE) **(15M)**

(a) Evaluate $\int \sin^{-1}\sqrt{x} \; dx$

(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(1+\cot x)} \, dx$

(c) Estimate $\int_0^4 x^2 \, dx$ using simpson's rule and n = 4.

(d) Solve the differential equation
$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

(e) Solve $dy/dx = 1 - y$; $y(0) = 0$, find $y(0.1)$ and $y(0.3)$ using Euler's method. Taking $h = 0.1$.

(f) Solve the differential equation
$(x + 1)\frac{dy}{dx} - y = e^x (x + 1)^2$

Q. 4 Attempt the following (Any THREE) **(15)**

(a) Show that $f(x, y) = 2x^2 + 3xy$ is continuous at $(2, 3)$

(b) Find the second order derivatives of $f(x,y)=x^2y^3 + x^4y$

(c) If $z=x^2y$, $x=t^2$ and $y=t^3$ Use chain rule to find $\frac{dz}{dt}$.

(d) Find the directional derivative of $f(x, y)=x^3 + 2xy^2$ at the point (-2, -3) in the direction of the vector $a= i + j$

(e) Find the gradient vector of $f(x, y)$ if $f(x, y) = 10 - 8x^2 - 2y^2$. Evaluate it at (2, 3)

(f) Find the equation for the tangent plane and parametric equations for normal line to the surface $z=x^2y$ at the point (2, 1, 4)

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Q. 5 Attempt the following (Any THREE) **(15)**

(a) Locate all relative extrema and saddle points of
$f(x, y)= 3x^2 - 2xy + y^2 - 8y$

(b) Solve the differential equation
$\frac{dy}{dx} = (4x + y + 1)^2$

(c) Draw the graph of $y= 4 - 3x^2 + x^3$ and find the intervals on which the function y is increasing and decreasing(draw the graph on the answer sheet itself)

(d) Find the asymptotes of the function $y= \frac{x}{(x+1)(x+2)^2}$

(e) Solve the differential equation
$dy/dx = (4x + y + 1)^2$

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