Q.P. Code :05600

[Time: $2^1/_2$ Hours] **[Marks:75]**

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 Answer following questions. **15**

1) Let f be defined on an interval, and let x_1 and x_2 be points on the interval, then f is said to be decreasing if **05**
   p) f $(x_1) > f(x_2)$ whenever $x_1 < x_2$
   q) $f(x_1) > f(x_2)$ whenever $x_1 > x_2$
   r) $f(x_1)= f(x_2)$ whenever $x_1 < x_2$
   s) None of these
2) If a function f is concave up on (a,b) then which of the following is true on (a,b)
   p) $f' > 0$
   q) $f' < 0$
   r) $f' = 0$
   s) None of these
3) If f is integrable on [a, b] and $f(x) \geq 0 \forall x \in [a,b]$, then
   p) $\int_a^b f(x) > 0$
   q) $\int_a^b f(x) \geq 0$
   r) $\int_a^b f(x) = 0$
   s) None of these
4) A rule that assigns a unique real number f (x,y) to each point (x,y) in some set D in the xy-plane is called
   p) a function of one variable
   q) a function of two variable
   r) a function of three variables
   s) None of these
5) which of the following is true about the function $f(x,y)=3x^2y^5$?
   p) Discontinuous at (0,0)
   q) Discontinuous at (1,1)
   r) Continuous everywhere
   s) None of these

b) Fill in the blanks for the following questions: **05**

1) A function f has a relative maximum at x_0 if there is an open interval containing x_0 on which f (x) is ---- f (x_0) for every x in the interval.
2) The points on the curve y=f(x) where the rate of change of y with respect to x changes from increasing to decreasing , or vice versa is known as-------
3) The integral $\int_0^{\pi} \sqrt{(1+cosx)^2}\, dx$ is the arc length of y= ----------- from x=0 to x=π.
4) If f (x,y) =$\frac{x-y}{x+y+1}$, the value of f (y+1,y) is given by ----------
5) The value of $\lim_{(x,y)\to(0,1)} e^{xy2}$=---------

(Turn Over)

c) State true or false for the following questions:

1) If f (x)=0 has a root, then Newtons Method starting at x=x_1 will approximate the root nearest x_1.
2) The order of the differential equation $(\frac{dy}{dx})^2 = \frac{dy}{dx}$ is one.
3) If f (x,y)→L as (x,y) approaches (0,0) along the x-axis ,and if f(x,y) →L as (x,y) approaches (0,0) along the y-axis then $\lim_{(x,y)\to(0,0)}$ f (x,y)=L.
4) If a function f is continuous at every point in an open set D, then f is continuous on D.
5) A function f of two variables is said to have a relative maximum at a point (x_0,y_0) if there is a disk centered at (x_0,y_0) such that $f(x_0,y_0) \leq f(x,y)$ for all points (x,y) that lie inside the disk.

Q.2 Answer any THREE of the following questions: **15**

a) Find the intervals on which $f(x) = x^2 - 3x+8$ is increasing and the intervals on which it is decreasing.
b) Use first and second derivative tests to show that $f(x) = 3x^2 - 6x + 1$ has a relative minimum at x=1.
c) Sketch the graph of the equation $y=x^3 - 3x + 2$ and identify the locations of the intercepts (draw the graph on the answer sheet itself).
d) Find the absolute maximum and minimum values of $f(x)= (x-2)^2$ in [1,4].
e) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?
f) The equation $x^3 - 2x - 2 = 0$ has one real solution. Approximate it by Newtons Method.

Q.3 Answer any THREE of the following questions: **15**

a) Find the area under the curve $y = x^3$ over the interval [2,3].
b) Find the area of the region bounded above by y = x+6, bounded below by $y = x^2$ and bounded on the sides by the lines x = 0 and x = 2.
c) Find the approximate value of $\int_1^2 \frac{1}{x}\,dx$ using Simpson's rule with n=10.
d) Solve differential equation $\frac{dy}{dx} = \frac{y}{x}$.
e) Use Euler's Method with a step size of 0.25 to find approximate solution of the initial-value problem $\frac{dy}{dx} = x - y^2, y(x) = 1$ over $0 \leq x \leq 1$.
f) Solve the differential equation $\frac{dy}{dx} + 3y = e^{-2x}$ by the method of integrating factors.

Q.4 Answer any THEREE of the following questions: **15**

a) If f (x,y)= $\frac{xy}{x^2+y^2}$, find the limit of f(x,y) as (x,y)→(0,0) 1) along x-axis and 2) along the line y=x.
b) Evaluate $\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}.\log(x^2+y^2)$,by converting to polar coordinates.
c) Find $f_x(x,y)$ and f_x(x,y) for f(x,y)=$2x^3y^2 + 2y + 4x$, and use those partial derivatives to compute $f_x(1,3)$ and $f_y(1,3)$.
d) Find the directional derivative of f(x,y)=e^{xy} at (2,0) in the direction of unit vector that makes an angle of $\frac{\pi}{3}$ with the positive x-axis.
e) Find an equation of the tangent plane to the surface $x^2 + 4y^2 + z^2 = 18$ at the point (1.,2,1). Also find the parametric equation of the line that is normal to the surface at the point (1,2,1).
f) Find all relative extrema and saddle points of f(x,y)=$3x^2 - 2xy + y^2 - 8y$.

(Turn Over)

Q.5 Answer any THREE of the following questions: **15**

a) Find the absolute maximum and minimum values of f (x) $=\frac{x-2}{x+2}$ on [-1,5].

b) Show that for any constants A and B, the function y= $Ae^{2x} + Be^{-4x}$ satisfies the equation y"+2y-8y=0.

c) Find the area of the region under the curve y=x^2+1 and over the interval [0,3].

d) Solve differential equation $\frac{dy}{dx} + 2xy = x.$

e) Determine whether the following limit exists. If so, find its value. $\lim_{(x,y)\to(0,0)} \frac{x^4-16y^4}{x^2+4y^2}$.