USCS105-DM

Q.P. Code : 781301

(2½ Hours) [Total Marks: 75]

N.B. (1) All questions are compulsory.
(2) Figures to the right indicate marks.

1. Answer the following questions **(15 M)**

(a) **Choose the best choice for the following questions:** **(5 M)**

(i) If f_1 and f_2 are two functions from R to R such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$, then $(f_1+f_2)(x)$ is given by
(a) x (b) x^2 (c) - x (d) None of these

(ii) Let {an} be a sequence that satisfies the recurrence relation $a_n = a_{n-1} + 3$ for n = 1, 2, 3, . . ., and suppose that $a_0 = 2$. What are a_1 and a_2?
(a) 5 and 8 respectively (b) 8 and 5 respectively
(c) 3 and 5 respectively (d) None of these

(iii) A class contains 10 students with 6 men and 4 women. Number of ways to elect a president, vice president, and treasurer is:
(a) 132 (b)122 (c) 120 (d) 121

(iv) There are four bus lines between *A* and *B*, and three bus lines between *B* and *C*. Number of ways that a man can travel by bus from *A* to *C* by way of *B* is
(a) 10 (b) 11 (c) 12 (d) 13

(v) An undirected graph with no multiple edges or loops is called
(a) tree (b) complex graph (c) simple graph (d) pseudo graph.

(b) **Fill in the blanks for the following questions:** **(5M)**

(i) A function f such that f(x) = x for any x in the domain of f is said to be a _____________ function.

(ii) A relation R on a set A is called ________ if whenever (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R, for all a, b, c ∈ A.

(iii) The Gödel number of a word $w = a_5a_2a_3a_1a_2$ is________

(iv) If a first task can be done in n_1 ways and a second task in n_2 ways, and if these tasks cannot be done at the same time then there are _________ ways to do either task.

(v) Let G be a directed graph and v be a vertex of G. The number of edges ending at v is called __________.

(c) **Answer the following questions:** **(5M)**

(i) Why is f, defined by f(x) = 1/(1-x), not a function from R to R?

(ii) Find the Fibonacci numbers f_2 and f_3.

(iii) State the essential difference between permutations and combinations, with examples.

(iv) State Product rule in counting of objects.

(v) What does it mean for a string to be derivable from a string ω by phase structure grammar G?

2. Answer any *three* of the following: (15 M)

(a) Let $S = \{-1, 0, 2, 4, 7\}$. Find $f(S)$ if (i) $f(x) = 1$, (ii) $f(x) = 2x + 1$.

(b) Define one-to-one function. Determine whether each of the following functions from {a, b, c, d} to itself is one-to-one.
i) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
ii) $f(a) = b, f(b) = b, f(c) = d, f(d) = c$

(c) Let R be the relation on the set of real numbers such that aRb if and only if $a - b$ is an integer. Is R an equivalence relation? Justify your answer.

(d) Define a poset. Is (S,R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if
i) a is no shorter than b?
ii) a weighs more than b?

(e) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$.

(f) Describe Tower of Hanoi puzzle. Formulate a recurrence relation for it.

3. Answer any *three* of the following: (15 M)

(a) How many ways in which 5 men and 5 women stand in a row so that no two men and no two women are adjacent to each other?

(b) State and prove Pascal identity.

(c) State Pigeonhole principle. A chess player has 77 days to prepare for a serious tournament. He decides to practice by playing at least one game per day and a total of 132 games. Show that there is a succession of days during which he must have played exactly 21 games.

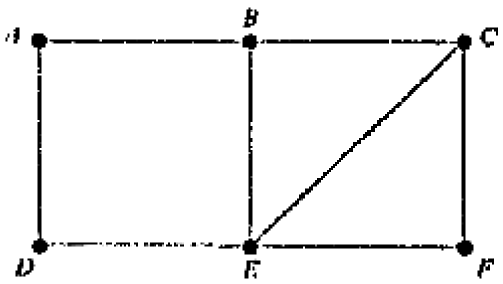
(d) How many integers between 1 and 600 (both inclusive) are not divisible by 3, 5 or 7?

(e) Define a language L over an alphabet A. Let A= {a, b, c}. Find L^* where language $L = \{a, b, c^3\}$

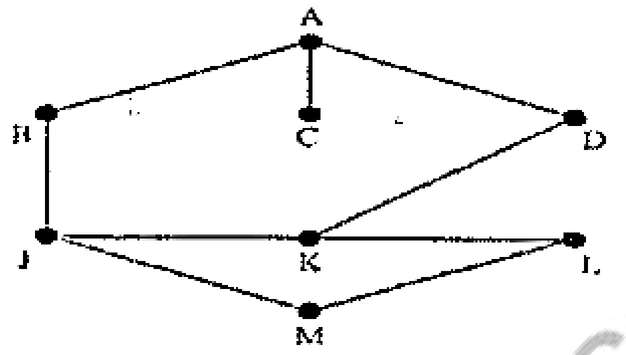
(f) Let $A = \{a, b\}$. Construct an automaton M which will accept precisely those words from A which ends in two b's.

4. Answer any *three* of the following: (15 M)

(a) Consider the graph G in the following figure. Find: (i) all cycles which include vertex A, (ii) all cycles in G.



(b) Consider the graph G in the following figure (where the vertices are ordered alphabetically). (i) Find the adjacency structure of G. (ii) Find the order in which the vertices of G are processed using a Breadth-first search algorithm beginning at vertex A.

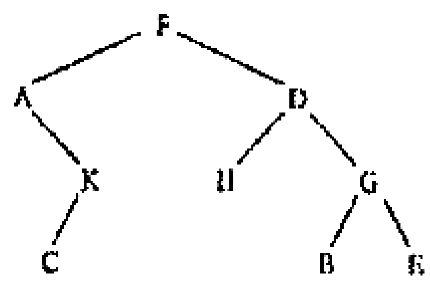


(c) Suppose a graph G contains two distinct paths from a vertex u to a vertex v. Show that G has a cycle.

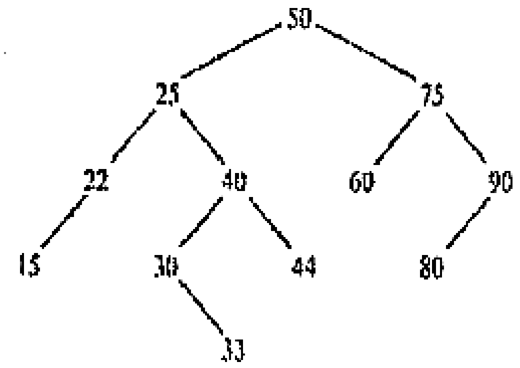
(d) Draw the graph G corresponding to each adjacency matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 2 & 0 & 2 & 2 \end{bmatrix}$$

(e) Consider the binary tree T in the following figure.
(i) Traverse T using the inorder algorithm.
(ii) Traverse T using the postorder algorithm.



(f) Let T be the binary search tree in the following figure. Suppose nodes 22, 25, 75 are deleted one after the other from T. Find the final tree T.



5. **Answer any *three* of the following:** (15 M)

(a) Draw the Hasse diagram for divisibility on the set {1, 2, 4, 8, 16, 32, 64}.

(b) How many solutions does the equation $x+y+z=11$ have, where x, y and z are non-negative integers with $x \geq 3$, $y \geq 1$ and $z \geq 0$?

(c) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3^n$.

(d) What is the coefficient of $x^{12}y^{13}$ in the expansion $(x+y)^{25}$ using binomial theorem.

(e) Draw all possible non similar binary trees *T* with three nodes.