

- N.B.** (1) All questions are compulsory.
 (2) Figures to the right indicate marks.

1. Answer the following questions**(15 M)****(a) Choose the best choice for the following questions:****(5 M)**

- (i) If f_1 and f_2 are two functions from $\mathbf{R}$ to $\mathbf{R}$ such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$, then $(f_1 \cdot f_2)(x)$ is given by
 (a) $x^3 + x^4$ (b) $x^3 - x^4$
 (c) $x^4 + x^3$ (d) None of these
- (ii) Which of the following is true about the poset $(\mathbf{Z}^+, |)$?
 (a) Zero is the least element (b) One is the least element
 (c) There is no least element (d) None of these
- (iii) A class contains 10 students with 6 men and 4 women. Number of ways to select a 4-member committee with 2 men and 2 women is given by
 (a) 60 (b) 70 (c) 80 (d) 90
- (iv) Suppose a bookcase shelf has 5 History texts, 3 Sociology texts, 6 Anthropology texts, and 4 Psychology texts. Number of ways a student can choose one text of each type is given by
 (a) 360 (b) 460 (c) 560 (d) 660
- (v) A loop is an edge connecting
 (a) a vertex with itself (b) two distinct vertices
 (c) no vertices (d) three distinct vertices

(b) Fill in the blanks for the following questions:**(5M)**

- (i) A function f is said to be strictly _____ if $f(x) > f(y)$ for any x and y in the domain of f .
- (ii) A relation R on a set A is called _____ if $(a, a) \in R$ for every element $a \in A$.
- (iii) The Gödel number of a word $w = a_3a_2a_1a_3a_4$ is _____
- (iv) Suppose that a procedure can be broken down into two tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are _____ ways to do the procedure.
- (v) Let G be a directed graph and v be a vertex of G . The number of edges beginning at v is called _____.

(c) Answer the following questions:**(5M)**

- (i) Why is f , defined by $f(x) = 1/(x+1)$, not a function from $\mathbf{R}$ to $\mathbf{R}$?
- (ii) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} + 2$ for $n = 1, 2, 3, \dots$ and suppose that $a_0 = 2$. What are a_1 and a_2 ?

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- (iii) State inclusion exclusion principle.
- (iv) Define a regular grammar.
- (v) Define a directed graph.

2. Answer any *three* of the following:

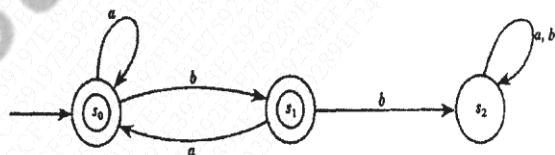
(15 M)

- (a) Find the domain and range of the following functions:
 - i) the function that assigns to each nonnegative integer its last digit.
 - ii) the function that assigns to a bit string the number of bits in the string.
- (b) Determine whether the function f from R to R defined by $f(x) = x - 2$ is bijective.
- (c) Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x . Is R an equivalence relation? Justify your answer.
- (d) Define Lattice. Determine whether the posets $(\{1, 2, 3, 4, 5\}, |)$ and $(\{1, 2, 4, 8, 16\}, |)$ are lattices.
- (e) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.
- (f) Define Fibonacci numbers. Formulate a recurrence relation for Fibonacci numbers.

3. Answer any *three* of the following:

(15 M)

- (a) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?
- (b) State and prove Vandermonde's identity.
- (c) State Pigeonhole principle. From any set of 13 integers, prove that there will be at least one pair which leaves the same remainder when divisible by 12.
- (d) How many integers between 1 and 600 (both inclusive) are not divisible by both 3 and 5?
- (e) Define a language L over an alphabet A . Let $A = \{a, b, c\}$. Find L^* where language $L = \{b^2\}$.
- (f) Determine whether or not the automaton M in the following figure accepts the words: $w_1 = ababba$; $w_2 = baab$; $w_3 = \lambda$ the empty word.

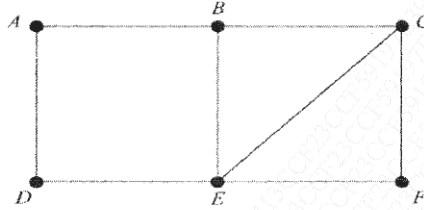


4. Answer any *three* of the following:

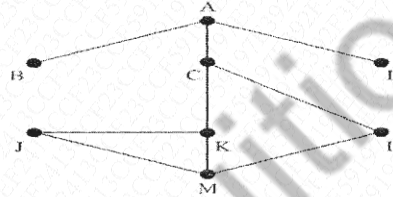
(15 M)

- (a) Consider the graph G in the following. Find (i) $\text{diam}(G)$, the diameter of G , (ii) $d(A, F)$, the distance from A to F .

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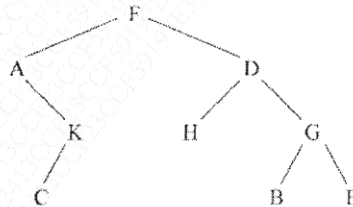
- (b) Consider the graph G in the following figure (where the vertices are ordered alphabetically). (i) Find the adjacency structure of G . (ii) Find the order in which the vertices of G are processed using a depth-first search algorithm beginning at vertex A .



- (c) Draw the graph G corresponding to each adjacency matrix:

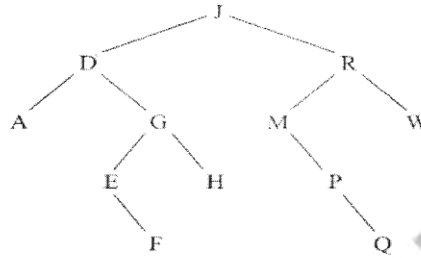
$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

- (d) Consider the binary tree T in the following figure. (i) Find the depth d of T . (ii) Traverse T using the post-order algorithm.



- (e) Suppose a graph G contains two distinct paths from a vertex u to a vertex v . Show that G has a cycle.
- (f) Consider the binary tree T in the following figure. Describe the tree T after (i) the node M and (ii) the node D are deleted.

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5. Answer any *three* of the following:

(15 M)

- Draw the Hasse diagram for divisibility on the set $\{1, 2, 3, 5, 7, 11, 13\}$.
- How many solutions does the equation $x+y+z=11$ have, where x, y and z are non-negative integers with $x \leq 3, y \leq 4$ and $z \leq 6$?
- Draw all possible non similar binary trees T with four external nodes.
- Show that $a_n = n \cdot 2^n$ is a solution of the non-homogeneous linear recurrence relation $a_n = 2a_{n-1} + 2^n$.
- What is the language generated by phase structure grammar G ?