

Solution Key Module 1

Q1) Find the extremals of $\int_{x_1}^{x_2} (1 + x^2 y') y' dx$?

Solution :

$$\because F = (1 + x^2 y') y'$$

here F does not contain y explicitly

So by formula

$$F = y' + x^2 y'^2$$

$$\therefore \frac{\partial F}{\partial y'} = 1 + 2x^2 y'$$

$$\because \frac{\partial F}{\partial y'} = C \quad (\text{formula})$$

$$1 + 2x^2 y' = C'$$

$$\therefore 2x^2 y' = C' - 1 = C$$

$$\therefore y' = \frac{C}{2x^2}$$

$$y' = \frac{C}{x^2}$$

Integrating both the sides

$$\int y' dx = \int \frac{C}{x^2} dx$$

$$y = -\frac{C}{x} + C_2$$

Changing the constants

$$y = \frac{C_1}{x} + C_2$$

The extremals of $\int_{x_1}^{x_2} (1 + x^2 y') y' dx$ is $y = \frac{C_1}{x} + C_2$

Q2) Find the extremal of the function $\int_0^{\pi/2} (y'^2 - y^2 + 2xy) dy$ with $y(0) = 0$; $y(\frac{\pi}{2}) = 0$

Solution: $\because F = y'^2 - y^2 + 2xy$

$$\frac{\partial F}{\partial y} = -2y + 2x$$

and $\because \frac{\partial F}{\partial y'} = 2y'$

Substituting values in Euler Equation

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0$$

$$-2y + 2x - \frac{d}{dx} (2y') = 0$$

$$\therefore \frac{d^2y}{dx^2} + y = x$$

The A.E is $D^2 + 1 = 0$

$$\therefore (D+1)(D-1) = 0$$

$$\therefore D = +i; -i$$

$$CF = y = c_1 \cos x + c_2 \sin x$$

$$PI = \frac{1}{D^2+1} \cdot x$$

$$= (D^2+1)^{-1} x$$

$$= (1 - D^2 + D^4 - \dots) x = x$$

The complete solution is $y = c_1 \cos x + c_2 \sin x + x$

when $x = 0$; $y = 0$

$$0 = c_1$$

when $x = \frac{\pi}{2}$; $y = 0$

$$0 = c_2 + \frac{\pi}{2}$$

$$\therefore C_2 = -\frac{\pi}{2}$$

$$\therefore y = x - \frac{\pi}{2} \sin x$$

The extremal of function is $y = x - \frac{\pi}{2} \sin x$.

Q3) Find the extremal of $\int_0^2 (xy + y^2 - 2y^2y') dx$

Solution:

$$\because F = xy + y^2 - 2y^2y'$$

$$\frac{\partial F}{\partial y} = x + 2y - 4yy'$$

$$\text{and } \frac{\partial F}{\partial y'} = -2y^2$$

Substituting these values in Euler's Equation

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0$$

$$\therefore x + 2y - 4yy' - \frac{d}{dx} (-2y^2) = 0$$

$$\therefore x + 2y - 4yy' + 4yy' = 0$$

$$\therefore x + 2y = 0$$

$$\therefore y = -\frac{x}{2}$$

\therefore The extremal of function is $y = -\frac{x}{2}$

Q4) Find the extremal of $\int_{x_0}^{x_1} (2xy - y'^2) dx$

Solution:

$$\because F = 2xy - y'^2$$

$$\frac{\partial F}{\partial y} = 2x$$

$$\frac{\partial f}{\partial y'} = 0 \quad \text{and} \quad \frac{\partial f}{\partial y''} = -2y''$$

$\therefore \frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) + \frac{d^2}{dx^2} \left(\frac{\partial f}{\partial y''} \right) = 0$ then the equation is extremum.

$$\therefore 2x - 0 + \frac{d^2}{dx^2} (-2y'') = 0$$

$$\therefore 2x - 2 \frac{d^2}{dx^2} \left(\frac{d^2 y}{dx^2} \right) = 0$$

$$\therefore 2x - 2 \frac{d^4 y}{dx^4} = 0$$

$$\therefore \frac{d^4 y}{dx^4} = x$$

The above equation is linear differential equation of fourth order.

$$\text{Its A.E is } D^4 = 0$$

$$D = 0, 0, 0, 0$$

$$PI = \frac{1}{D^4} x$$

$$\therefore y = \frac{1}{D^3} \int x dx = \frac{1}{D^3} \cdot \frac{x^2}{2}$$

$$\therefore = \frac{1}{D^2} \int \frac{x^2}{2} dx = \frac{1}{D^2} \cdot \frac{x^3}{6}$$

$$\therefore = \frac{1}{D} \int \frac{x^3}{6} dx = \frac{1}{D} \frac{x^4}{24}$$

$$\therefore = \int \frac{x^4}{24} dx$$

$$y = \frac{x^5}{120}$$

Hence $y = C_1 + C_2x + C_3x^2 + C_4x^3 + \frac{x^5}{120}$ is the solution.

Q5). Using Rayleigh-Ritz Method solve the boundary value problem $I = \int_0^1 (xy + \frac{1}{2}y'^2) dx$; $0 \leq x \leq 1$ given $y(0) = 0$ and $y(1) = 0$?

Solution: $F = xy + \frac{1}{2}y'^2$

Assuming Trial solution

$$\bar{y}(x) = C_0 + C_1x + C_2x^2$$

$$\bar{y}(0) = 0$$

$$\therefore C_0 = 0$$

$$\bar{y}(1) = 0$$

$$\therefore C_1 + C_2 = 0$$

$$\therefore C_2 = -C_1$$

$$\bar{y}(x) = C_1x - C_1x^2 = C_1(x - x^2)$$

$$\bar{y}'(x) = C_1(1 - 2x)$$

Putting values in $I = \int_0^1 (xy + \frac{1}{2}y'^2) dx$

$$I = \int_0^1 \left\{ x[C_1(x - x^2)] + \frac{1}{2}C_1^2(1 - 2x)^2 \right\} dx$$

$$= \int_0^1 [C_1(x^2 - x^3) + \frac{1}{2}C_1^2(1 - 4x + 4x^2)] dx$$

$$= C_1 \left[\int_0^1 x^2 dx - \int_0^1 x^3 dx \right] + \frac{1}{2}C_1^2 \left[\int_0^1 (1 - 4x + 4x^2) dx \right]$$

$$= G \left\{ \left(\frac{x^3}{3} \right)'_0 - \left(\frac{x^4}{4} \right)'_0 \right\} + \frac{1}{2} G^2 \left\{ (x)'_0 - 4 \left(\frac{x^2}{2} \right)'_0 + 4 \left(\frac{x^3}{3} \right)'_0 \right\}$$

$$= G \left\{ \frac{1}{3} - \frac{1}{4} \right\} + \frac{1}{2} G^2 \left\{ 1 - 2 + \frac{4}{3} \right\}$$

$$\therefore = G \left(\frac{1}{12} \right) + \frac{1}{2} G^2 \left(\frac{1}{3} \right)$$

$$I = \frac{G}{12} + \frac{G^2}{6}$$

Stationary values are

$$\frac{dI}{dG} = 0$$

$$\therefore \frac{1}{12} + \frac{2G}{6} = 0$$

$$\therefore G = -\frac{1}{4}$$

Putting values in eqn (1)

$$\bar{y}(x) = \frac{1}{4}(x - x^2)$$

$$\bar{y}(x) = \frac{1}{4}x(x-1)$$

Q6) Using Rayleigh - Ritz Method solve the boundary value problem

$$I = \int_0^1 (xy + \frac{1}{2}y'^2) dx \quad 0 \leq x \leq 1$$

given $y(0) = 0$ and also $y(1) = 0$?

Solution : Extremise $I = \int_0^1 (xy + \frac{1}{2}y'^2) dx$

$$\text{where } F = xy + \frac{1}{2}y'^2$$

Assume trial solution $\bar{y}(x) = C_0 + C_1x + C_2x^2$ — (3)

Given data

$$\bar{y}(0) = 0$$

$$\therefore C_0 = 0 \text{ and } \bar{y}(1) = 0$$

$$\therefore C_1 + C_2 = 0$$

$$\therefore C_2 = -C_1$$

$$\therefore \bar{y}(x) = C_1x - C_1x^2$$

$$\bar{y}(x) = C_1(x - x^2) \text{ — (4)}$$

$$\bar{y}'(x) = C_1(1 - 2x)$$

Putting values in eqn 1

$$I = \int_0^1 (xy + \frac{1}{2}y'^2) dx$$

$$I = \int_0^1 \left\{ x[C_1(x - x^2)] + \frac{1}{2}C_1^2(1 - 2x)^2 \right\} dx$$

$$= \int_0^1 [C_1(x^2 - x^3) + \frac{1}{2}C_1^2(1 - 4x + 4x^2)] dx$$

$$= C_1 \left[\int_0^1 x^2 dx - \int_0^1 x^3 dx \right] + \frac{C_1^2}{2} \left[\int_0^1 1 dx - 4 \int_0^1 x dx + 4 \int_0^1 x^2 dx \right]$$

$$= C_1 \left\{ \left[\frac{x^3}{3} \right]_0^1 - \left[\frac{x^4}{4} \right]_0^1 \right\} + \frac{C_1^2}{2} \left\{ \left[x \right]_0^1 - 4 \left[\frac{x}{2} \right]_0^1 + 4 \left[\frac{x^3}{3} \right]_0^1 \right\}$$

$$= C_1 \left[\frac{1}{3} - \frac{1}{4} \right] + \frac{1}{2}C_1^2 \left[1 - 2 + \frac{4}{3} \right]$$

$$\therefore I = C_1 \left(\frac{1}{12} \right) + \frac{1}{2}C_1^2 \left(\frac{1}{3} \right)$$

$$\therefore I = \frac{G}{12} + \frac{C_1^2}{6}$$

Its Stationary Values are given by

$$\frac{dI}{dG} = 0$$

$$\therefore \frac{1}{12} + \frac{2G}{6} = 0$$

$$\therefore G = -\frac{1}{4}$$

Putting this value in eqn 4

$$\bar{y}(x) = -\frac{1}{4}(x - x^2)$$

$$\bar{y}(x) = \frac{1}{4}(x-1)x$$

$$\therefore \bar{y}(x) = \frac{1}{4}x(x-1)$$

Last Moment Tutorials

Solution Key Module 2

Q1) Verify Cauchy-Schwarz inequality for the vectors
 $u = (-4, 2, 1)$ and $v = (8, -4, -2)$

Solution: $\|u\| = \sqrt{16+4+1}$ $\|v\| = \sqrt{64+16+4}$
 $\|u\| = \sqrt{21}$ $\|v\| = \sqrt{84}$

$$\therefore \|u\| \|v\| = \sqrt{21} \sqrt{84} = \sqrt{21} \sqrt{21} \sqrt{4} = 42$$

$$\begin{aligned} |u \cdot v| &= |u_1 v_1 + u_2 v_2 + u_3 v_3| \\ &= |(-4)(8) + 2(-4) + 1(-2)| \\ &= |-32 - 8 - 2| \\ &= 42 \end{aligned}$$

Hence $\|u\| \|v\| = |u \cdot v|$

Hence Cauchy-Schwarz inequality holds good for given vectors

Q2) Find a unit vector orthogonal to both $(1, 1, 0)$ and $(0, 1, 1)$

Solution: Let the required vector be (v_1, v_2, v_3)

Since it's orthogonal to both u_1 & u_2

$$u_1 \cdot v = (1, 1, 0) \cdot (v_1, v_2, v_3) = v_1 + v_2 + 0 = 0$$

$$u_2 \cdot v = (0, 1, 1) \cdot (v_1, v_2, v_3) = 0 + v_2 + v_3 = 0$$

By Cramer's Rule

$$\frac{v_1}{1-0} = \frac{-v_2}{1-0} = \frac{v_3}{1-0} = k$$

$$\therefore v_1 = k ; v_2 = -k ; v_3 = k$$

Since the normal is 1

$$v_1^2 + v_2^2 + v_3^2 = 1$$

$$\therefore k^2 + k^2 + k^2 = 1$$

$$\therefore k = \pm \frac{1}{\sqrt{3}}$$

When $k = \frac{1}{\sqrt{3}}$

$$v_1 = \frac{1}{\sqrt{3}} ; v_2 = \frac{-1}{\sqrt{3}} ; v_3 = \frac{1}{\sqrt{3}}$$

When $k = \frac{-1}{\sqrt{3}}$

$$v_1 = \frac{-1}{\sqrt{3}} ; v_2 = \frac{1}{\sqrt{3}} ; v_3 = \frac{-1}{\sqrt{3}}$$

Q3) Determine whether the set of vectors of the form (a, b, c) where $b = a + c$ form a subspace of R^3 under usual addition and scalar multiplication?

Solution: Let $v_1 = (a_1, b_1, c_1)$ and $v_2 = (a_2, b_2, c_2)$ be two vectors in R^3

Now, $v_1 + v_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

Since $b_1 = a_1 + c_1$ and $b_2 = a_2 + c_2$

$$\begin{aligned} b_1 + b_2 &= (a_1 + c_1) + (a_2 + c_2) \\ &= a_1 + a_2 + c_1 + c_2 \end{aligned}$$

$\therefore u + v$ is of form (α, β, γ) and $\beta = \alpha + \gamma$

$\therefore u + v$ is in R^3

If k is a scalar then $kV_1 = (ka_1, kb_1, kc_1)$

Since $b_1 = a_1 + c_1$

$$kb_1 = k(a_1 + c_1) = ka_1 + kc_1$$

$\therefore kV_1$ is of form (p, q, r) and $q = p + r$

$\therefore kV_1$ is in R^3 .

Hence the set of vectors of form (a, b, c) is subspace of R^3 .

Q4) Express the following as a linear combination of

$$V_1 = (2, 1, 4) \quad V_2 = (1, -1, 3) \quad \text{and} \quad V_3 = (3, 2, 5)$$

a) $(6, 11, 6)$?

Solution : a) $(6, 11, 6)$

$$\text{Let } W = (6, 11, 6)$$

Find k_1, k_2, k_3 such that

$$W = k_1V_1 + k_2V_2 + k_3V_3$$

$$(6, 11, 6) = k_1(2, 1, 4) + k_2(1, -1, 3) + k_3(3, 2, 5)$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}$$

$$\Rightarrow k_1 = 4$$

$$k_2 = -5$$

$$k_3 = 1$$

$$\therefore W = 4V_1 - 5V_2 + (1)V_3$$

$$(6, 11, 6) = 4(2, 1, 4) - 5(1, -1, 3) + 1(3, 2, 5)$$

Q5) Express $p(x) = 7 + 8x + 9x^2$ as linear combination of
 $P_1 = 2 + x + 4x^2$; $P_2 = 1 - x + 3x^2$; $P_3 = 2 + x + 5x^2$

Solution: let $p = k_1 P_1 + k_2 P_2 + k_3 P_3$

$$\begin{aligned} 7 + 8x + 9x^2 &= k_1(2 + x + 4x^2) + k_2(1 - x + 3x^2) + \\ &\quad k_3(2 + x + 5x^2) \\ &= (2k_1 + k_2 + 2k_3) + (k_1 - k_2 + k_3)x + (4k_1 + 3k_2 + 5k_3)x^2 \end{aligned}$$

Solve equations

$$\begin{array}{rcl} 2k_1 + k_2 + 2k_3 = 7 & \text{---} & 1 \\ k_1 - k_2 + k_3 = 8 & \text{---} & 2 \\ 4k_1 + 3k_2 + 5k_3 = 9 & \text{---} & 3 \end{array}$$

taking eqn 1st as 2

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 4 & 3 & 5 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 9 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -9 \\ -5 \end{bmatrix}$$

By R_{23}

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -9 \end{bmatrix}$$

$$\therefore k_1 - k_2 + k_3 = 8 \quad ; \quad k_2 + k_3 = -5$$

$$3k_2 = -9$$

$$\therefore k_2 = -3$$

$$\therefore k_3 = -2$$

$$k_1 = 8 + k_2 - k_3$$

$$\therefore k_1 = 7$$

Hence

$$7 + 8x + 9x^2 = 7(2 + x + 4x^2) - 3(1 - x + 3x^2) - 2(2 + x + 5x^2)$$

Q6) Let R^4 have the Euclidean inner product. Express $w = (-1, 2, 6, 0)$ in the form $w = w_1 + w_2$ where $w_1 \in W$ and $w_2 \in W^\perp$ where W is spanned by $v_1 = (-1, 0, 1, 2)$ and $v_2 = (0, 1, 0, 1)$?

Solution: The project of w on W

$$\text{Proj}_W w = \langle w, v_1 \rangle v_1 + \langle w, v_2 \rangle v_2$$

$$\therefore \text{Proj}_W w = [(-1, 2, 6, 0) \cdot (-1, 0, 1, 2)] v_1 + [(-1, 2, 6, 0) \cdot (0, 1, 0, 1)] v_2$$

$$= 7v_1 + 2v_2$$

$$= 7(-1, 0, 1, 2) + 2(0, 1, 0, 1)$$

$$\therefore w_1 = (-7, 2, 7, 16)$$

The component of w orthogonal to W is

$$\text{Proj}_{W^\perp} w = w - \text{Proj}_W w$$

$$= (-1, 2, 6, 0) - (-7, 2, 7, 16)$$

$$\therefore w_2 = (6, 0, -1, -16)$$

$$\therefore w = w_1 + w_2 \quad \text{where } w_1 \in W \text{ and } w_2 \in W^\perp$$

$$w = (-7, 2, 7, 16) + (6, 0, -1, -16)$$

Q7). Let \mathbb{R}^3 have the Euclidean inner product. Use Gram Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis where $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$, $u_3 = (1, 2, 1)$

Solution: Step 1: $v_1 = u_1 = (1, 1, 1)$

$$\begin{aligned}\text{Step 2: } v_2 &= u_2 - \text{proj}_{v_1} u_2 \\ &= u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} \cdot v_1\end{aligned}$$

$$\langle u_2, v_1 \rangle = -1 + 1 - 0 = 0$$

$$\|v_1\|^2 = 1 + 1 + 1 = 3$$

$$\therefore v_2 = (-1, 1, 0) - \frac{0}{3}(1, 1, 1)$$

$$v_2 = (-1, 1, 0)$$

$$\text{Step 3: } v_3 = u_3 - \text{proj}_{v_1} u_3 - \text{proj}_{v_2} u_3 = u_3 - \frac{\langle u_3, v_1 \rangle}{\|v_1\|^2} \cdot v_1 - \frac{\langle u_3, v_2 \rangle}{\|v_2\|^2} \cdot v_2$$

$$\langle u_3, v_1 \rangle = 1 + 2 + 1 = 4$$

$$\langle u_3, v_2 \rangle = -1 + 2 + 0 = 1$$

$$\|v_1\|^2 = 3 \text{ as before}$$

$$\|v_2\|^2 = 1 + 1 + 0 = 2$$

$$\therefore v_3 = (1, 2, 1) - \frac{4}{3}(1, 1, 1) - \frac{1}{2}(-1, 1, 0)$$

$$v_3 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{3}\right)$$

Hence, $v_1 = (1, 1, 1)$; $v_2 = (-1, 1, 0)$ $v_3 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{3}\right)$

from orthogonal basis for \mathbb{R}^3

the norms of these vectors are

$$\|v_1\| = \sqrt{1+1+1} = \sqrt{3};$$

$$\|v_2\| = \sqrt{1+1+0} = \sqrt{2};$$

$$\|v_3\| = \sqrt{\frac{1}{36} + \frac{1}{36} + \frac{1}{9}} = \sqrt{\frac{6}{36}} = \frac{1}{\sqrt{6}}$$

Hence, the orthogonal basis for \mathbb{R}^3 is

$$q_1 = \frac{v_1}{\|v_1\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

$$q_2 = \frac{v_2}{\|v_2\|} = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$$

$$q_3 = \frac{v_3}{\|v_3\|} = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \right)$$

Q8) Let \mathbb{R}^4 have the Euclidean inner product. Use Gram-Schmidt Process to transform the basis $\{u_1, u_2, u_3\}$ into orthogonal basis where $u_1 = \{1, 0, 1, 1\}$, $u_2 = \{-1, 0, -1, 1\}$, $u_3 = \{0, -1, 1, 1\}$

Solution: Step 1: $v_1 = u_1 = \{1, 0, 1, 1\}$

$$\text{Step 2: } v_2 = u_2 - \text{Proj}_{v_1} u_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} \cdot v_1$$

$$\langle u_2, v_1 \rangle = -1 + 0 - 1 = -2$$

$$\langle u_2, v_1 \rangle = -2$$

$$\|v_1\|^2 = 1 + 0 + 1 + 1 = 3$$

$$\therefore v_2 = (1, 0, 1, 1) - \frac{1}{3} (1, 0, 1, 1)$$

$$v_2 = \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{4}{3}\right)$$

Step 3 $v_3 = u_3 - \text{proj } u_3$

$$u_3 - \frac{\langle u_3, v_1 \rangle}{\|v_1\|^2} \cdot v_1 - \frac{\langle u_3, v_2 \rangle}{\|v_2\|^2} \cdot v_2$$

$$\langle u_3, v_1 \rangle = 0 + 0 + 1 + 1 = 2$$

$$\begin{aligned} \langle u_3, v_2 \rangle &= (0, 1, 1, 1) \cdot \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{4}{3}\right) \\ &= 0 + 0 - \frac{2}{3} + \frac{4}{3} = \frac{2}{3} \end{aligned}$$

$$\|v_1\|^2 = 3$$

$$\|v_2\|^2 = \frac{4}{9} + 0 + \frac{4}{9} + \frac{16}{9}$$

$$\|v_2\|^2 = \frac{8}{3}$$

$$\therefore v_3 = (0, 1, 1, 1) - \frac{2}{3} (1, 0, 1, 1) - \frac{2/3}{8/3} \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{4}{3}\right)$$

$$v_3 = (0, 1, 1, 1) - \frac{2}{3} (1, 0, 1, 1) - \frac{1}{4} \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{4}{3}\right)$$

$$= (0, 1, 1, 1) + \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{2}{3}\right) + \left(\frac{1}{6}, 0, \frac{1}{6}, \frac{1}{3}\right)$$

$$v_3 = \left(\frac{1}{2}, 1, \frac{1}{2}, 0\right)$$

Hence the orthogonal set is

$$v_1 = (1, 0, 1, 1) \quad v_2 = \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{4}{3}\right) \quad v_3 = \left(\frac{1}{2}, 1, \frac{1}{2}, 0\right)$$

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