Paper / Subject Code: 40801 / Applied Mathematics-IV

S.E-EXTC-SEM-IV (Choice Based)

(3 hours) Total Marks:80

N.B: (1) Question no.1 is **compulsory.**

(2)Attempt any **three** questions from remaining **five** questions.

(3)**Figures** to the **right** indicate **full** marks.

(4)Assume suitable data if necessary.

1. (a) if $A = \begin{pmatrix} 2 & 4 \\ 0 & 3 \end{pmatrix}$, then find the eigen values of $6A^{-1} + A^3 + 2I$. (5)

(b) Find a vector orthogonal to both $u=(-6,4,2), v=(3,1,5)$. (5)

(c) Show that $\oint_C \log z\, dz = 2\pi i$, Where C is the unit circle in the Z-plane. (5)

(d) Let X be continuous random variable with probability distribution

$$p(X=x)=\begin{cases} \frac{x}{6}+k, & if\ 0\le x\le 3 \\ 0 & , elsewhere \end{cases}$$

Evaluate k and find $p(1\le x\le 2)$. (5)

2. (a) Show that the matrix A is diagonalizable. Also find the transforming matrix M and the diagonal matrix D where $A=\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. (6)

(b) Find the extremals of the function $\int_0^{\frac{\pi}{2}}\left((y')^2 - y^2 + 2xy\right)dx$ with $y(0)=0, y\left(\frac{\pi}{2}\right)=0$. (6)

(c) Let R^4 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ in to an orthonormal basis where $u_1=(1,0,1,1)$, $u_2=(-1,0,-1,1)$, $u_3=(0,-1,1,1)$. (8)

3. (a) The number of accidents in a year attributed to taxi driver in a city follows poisson distribution with mean 3. Out of 1,000 taxi drivers, find approximately the number of drivers with (i) No accident in a year (ii) more than 3 accident in a year. $(Given\ e^{-1}=0.3679, e^{-2}=0.1353, e^{-3}=0.0498)$ (6)

(b) Calculate Rank Correlation co-efficient for the following data: (6)

X:	10	12	18	18	15	40
Y:	12	18	25	25	50	25

(c) Expand $f(z)=\dfrac{1}{z^2(z-1)(z+2)}$ about $z=0$ for

$(i)\ |z|\le 1 \quad (ii)\ 1\le |z|\le 2 \quad (iii)\ |z|>2.$ **(8)**

4.(a) Evaluate $\oint_C \dfrac{\sin^6 z}{\left(z-\dfrac{\pi}{6}\right)^3}dz$, where C is the circle $|z|=1$. **(6)**

(b) Find the m.g.f. of a random variable whose probability density function is

$$p(X=x)=\begin{cases}\left(\dfrac{1}{2}\right)^x, & x=1,2,3\ldots\ldots \\ 0, & elsewhere\end{cases}$$

Hence, find the mean and variance. **(6)**

(c) Verify the Cayley-Hamilton Theorem for matrix A and hence find A^{-1} for .
$A=\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$, Hence, find $A^5-4A^4-7A^3+11A^2-A-10I$ in terms of A. **(8)**

5.(a) Express $p(x)=7+8x+9x^2$ as a linear combination of $p_1(x)=2+x+4x^2$, $p_2(x)=1-x+3x^2$, $p_3(x)=2+x+5x^2$. **(6)**

(b) Using Cauchy residue theorem, evaluate $\int_0^{2\pi}\dfrac{\cos 2\theta}{5+4\cos\theta}d\theta$ **(6)**

(c) In an examination marks obtained by students in mathematics, physics and chemistry are normally distributed with mean 51, 53, 46 and with standard deviation 15, 12, 16 respectively, find the probability of securing total marks (i) 180 or above (ii) 90 or below. **(8)**

6. (a) If $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$, Find A^{50}. **(6)**

(b) Using Cauchy residue theorem, evaluate $\oint_C \dfrac{e^z}{(z^2+\pi^2)^2}dz$, where C is the circle $|z|=4$. **(6)**

(c) Using Rayleigh-Ritz method, solve the boundary value problem
$I=\int_0^1\left(xy+\dfrac{1}{2}y'^2\right)dx;\ 0\le x\le 1$, given $y(0)=y(1)=0$ where $\bar{y}(x)=c_0+c_1x+c_2x^2$. **(8)**

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