Paper / Subject Code: 39601 / Applied Mathematics-IV

S.E - MECH - SEM - IV (Choice Based)

Q.P. Code: 37729

(3 hours) [Total marks: 80]

19 NOV 2018

N.B. (1) Question No. 1 is compulsory.

(2) Answer **any Three** from remaining

(3) Figures to the right indicate full marks

(4) Use of statistical tables is allowed.

1.a) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ is non derogatory 5

b) Determine all basic solutions to the following problem-

Maximize $z = x_1 + 3x_2 + 3x_3$

Subject to

$x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

$x_1, x_2, x_3, \geq 0$ 5

c) Prove that $\bar{F} = (2xy + z)i + (x^2 + 2yz^3)j + (3y^2z^2 + x)k$ is an irrotational vector and find the corresponding scalar ϕ such that $\bar{F} = \nabla\phi$. 5

d)Can it be concluded that the average lifespan of an Indian is more than 70 years if a random sample of 100 Indians has an average lifespan of 71.8 years with standard deviation of 8.9 years? 5

2.a)Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable. Also find the transforming matrix and the diagonal matrix. 6

b Using Green's Theorem ,evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = x^2$ and $y = \sqrt{x}$. 6

c)Solve the following problem by simplex method

Maximize $Z = 3x_1 + 2x_2$

Subject to $3x_1 + 2x_2 \leq 18,$

$x_1 \leq 4,$

$x_2 \leq 6, \quad x_1, x_2 \geq 0$ 8

3. a) Use Stoke's theorem evaluate $\int \bar{F}.d\bar{r}$ $\bar{F} = 2y(1-x)i + (x - x^2 - y^2)j + (x^2 + y^2 + z^2)k$ where and s is the surface of the plane $x+y+z = 2$ which is in the first octant 6

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b) The standard deviations calculated from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regard as drawn from the normal populations with the same standard deviation? (Given $F_{0.025}$ =3.51 with d.o.f. 8 and 12 and $F_{0.025}$ =4.20with d.o.f. 12 and 8 or $F_{0.05}$ =4.50 with d.o.f 8and12) 6

c) Use Penalty Method (Big M method) to solve the following L.P.P.

Minimize $z = 6x_1 + 4x_2$

Subject to the constraints

$2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3, \quad x_1, x_2 \geq 0$ 8

4.a)Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Hence find A^{-1} 6

b) Marks obtained by students in an examination follow normal distributions. If 30% of students got below 35 marks and 10% got above 60 marks. Find mean and standard deviation. 6

c) Use the dual simplex method to solve the following L.P.P.

Minimize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 2$, $-x_1 - x_2 \geq 1$, $x_1, x_2 \geq 0$. 8

5.a) Find e^A and 4^A, if $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ 6

b)A random discrete variable x has the probability density function given

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

find k, the mean and the variance 6

c) In an experiment on immunizations of cattle from Tuberculosis the following results were obtained. Use χ^2-test to determine the efficiency of vaccine in preventing tuberculosis. 8

	Affected	Not affected	Total
Inoculated	290	110	400
Not Inoculated	310	90	400
Total	600	200	800

6.a) Use Gauss divergence theorem to evaluate $\iint \bar{F}.\bar{N}\, ds$ where $\bar{F} = x^2\bar{i} + z\bar{j} + yz\bar{k}$ and s is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1 6

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b) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviation from the mean are 26.94 and 18.73 respectively. Can the sample be considered to have drawn from the same normal population? 6

c) Reduce the quadratic form, $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1$ to the canonical form.. Also find its rank, index and signature, using congruent transformations. 8

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