

SE - EXTC Sem - III CBCS choice based

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

20 NOV 2018

Q.1 (a) Evaluate  $\int_0^{\infty} e^{-2t} t \sin t \, dt$ . (5)

(b) Find  $a, b, c, d, e$  if  $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2) + i(dx^2y - 2y^3 + exy)$  is analytic. (5)

(c) Find half range sine series of  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$ . (5)

(d) Find directional derivative of  $\phi = 4xz^2 + x^2yz$ , at  $(1, -2, -1)$  in direction of  $2i - j - 2k$ . (5)

Q.2 (a) Prove that  $\nabla r^n = nr^{(n-2)} \vec{r}$ . (6)

(b) Find Bilinear Transformation which maps the points  $z = -1, 0, 1$  onto the points  $w = -1, -i, 1$ . (6)

(c) Find i)  $L^{-1} \left[ \frac{e^{-2s}}{s^2 + 3s + 2} \right]$  ii)  $L^{-1} \left[ \log \left( \frac{s^2 + 4}{s + 4} \right) \right]$ . (8)

Q.3 (a) Use Gauss's Divergence Theorem to evaluate  $\iiint \vec{N} \cdot \vec{F} \, ds$  where (6)

$\vec{F} = 4xi - 2y^2j + z^2k$  and  $s$  is region bounded by  $x^2 + y^2 = 4$ ,  $z = 0$ ,  $z = 4$ .

(b) Find Laplace Transform of  $e^{-2t} \int_0^t u e^{3u} \cos 4u \, du$ . (6)

(c) Obtain Fourier series of  $f(x) = \begin{cases} x + \frac{\pi}{2} & -\pi < x < 0 \\ \frac{\pi}{2} - x & 0 < x < \pi \end{cases}$ . (8)

Hence deduce  $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$

Q.4 (a) Show that set of functions  $\left\{ \frac{\sin x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \frac{\sin 3x}{\sqrt{\pi}}, \dots \right\}$  form an (6)

orthonormal set in  $(-\pi, \pi)$ .

(b) Find orthogonal trajectories of the family of curves  $e^x \cos y - xy = c$ . (6)

(c) Prove that  $\vec{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$  is irrotational. (8)  
Find scalar potential of  $\vec{F}$ . Hence find the work done of moving particle from (1,0,2) to (0,1,1).

Q.5 (a) Find Fourier Integral representation for  $f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$ . (6)

(b) Solve using Laplace Transform  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}$  given  $y(0) = 4$  and  $y'(0) = 2$ . (6)

(c) Verify Green's Theorem for  $\vec{F} = x^2i - xyj$  and  $c$  is triangle having vertices  $A(0,2)$ ,  $B(2,0)$ ,  $C(4,2)$ . (8)

Q.6 (a) Using Convolution theorem, find Inverse Laplace of  $\frac{s}{(s^2 + 4)^2}$ . (6)

(b) Prove that  $J_{-\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[ \frac{3}{x} \sin x + \frac{(3-x^2)}{x^2} \cos x \right]$ . (6)

(c) Find Fourier series for  $f(x) = (\pi - x)^2$  in  $0 \leq x \leq 2\pi$ . Hence deduce (8)  
that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$