

Sem - 3 Mech CBCS

Q. P. Code: 25565

(3hours)



[Total marks: 80]

20 NOV 2018

- N.B.** 1) Question No. 1 is compulsory.
 2) Answer **any Three** from remaining
 3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = \int_0^t u e^{-3u} \sin u du$. 5
- b) Show that the set of functions $\{\cos nx, n = 1, 2, 3 \dots\}$ is orthogonal on $(0, 2\pi)$. 5
- c) Does there exist an analytic function whose real part is $u = k(1 + \cos \theta)$? Give justification. 5
- d) The equations of lines of regression are $x + 6y = 6$ and $3x + 2y = 10$. Find
 i) means of x and y , ii) coefficient of correlation between x and y . 5
2. a) Evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$. 6
- b) Find the image of the triangle bounded by lines $x = 0, y = 0, x + y = 1$ in the z -plane under the transformation $w = e^{i\pi/4} z$. 6
- c) Obtain Fourier series of $f(x) = x^2$ in $(0, 2\pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$
3. a) Find the inverse Laplace transform of $F(s) = \frac{s}{(s^2+4)^2}$. 6
- b) Solve $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$, with $u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1 - x)$
 taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender – Schmidt method. 6
- c) Using Residue theorem, evaluate
- i) $\int_0^{2\pi} \frac{d\theta}{5 - 4\cos \theta}$ ii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2}$ 8

[TURN OVER]

4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,
 $u(0, t) = 0$, $u(5, t) = 100$, $u(x, 0) = 20$ taking $h = 1$ for one-time step. 6
- b) Obtain the Taylor's and Laurent series which represent the function
 $f(z) = \frac{z-1}{z^2-2z-3}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 3$ 6
- c) Solve $(D^2 + 4D + 8)y = 1$ with $y(0) = 0$ and $y'(0) = 1$ where $D \equiv \frac{d}{dt}$ 8
5. a) Find an analytic function $f(z) = u + iv$, if
 $u = e^{-x}\{(x^2 - y^2) \cos y + 2xy \sin y\}$ 6
- b) Find the Laplace transform of
 $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ and $f(t+2) = f(t)$ for $t > 0$. 6
- c) Obtain half range Fourier cosine series of $f(x) = x$, $0 < x < 2$. Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$
6. a) If $f(a) = \int_C \frac{4z^2+z+4}{z-a} dz$ where C is the ellipse $4x^2 + 9y^2 = 36$.
 Find, i) $f(4)$ ii) $f'(-1)$ and iii) $f''(-i)$ 6
- b) Use least square regression to fit a straight line to the following data, 6
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|---|----|----|----|----|----|----|----|----|----|----|
| x | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| y | 17 | 24 | 31 | 33 | 37 | 37 | 40 | 40 | 42 | 41 |
- c) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in form $y = a \sin(\pi x / l)$ from which it is released at a time $t = 0$. If the vibrations of a string is given by $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, show that the displacement of a point at a distance x from one end at time t is given by
 $y(x, t) = a \sin(\pi x / l) \cos(\pi c t / l)$ 8