

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

23 NOV 2017

Q 1. a) Evaluate the Laplace transform of $\sinh\left(\frac{t}{2}\right)\sin\left(\frac{\sqrt{3}}{2}t\right)$. [5]

b) Determine the constants a,b,c,d so that the function $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ is analytic. [5]

c) Find a unit normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. [5]

d) Obtain half range sine series for $f(x) = x$, $0 < x < 2$. [5]

Q 2. a) If $u = e^{2x}(x \cos 2y - y \sin 2y)$ then find analytic function $f(z)$ by Milne Thomson Method [6]

b) Find the Fourier series for $f(x) = 9 - x^2$, $-3 \leq x \leq 3$ [6]

c) Find the Laplace transform of the following

i) $L[\sqrt{1 + \sin t}]$ ii) $L\left[\frac{\sinh 2t}{t}\right]$ [8]

Q 3. a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ [6]

b) Evaluate inverse Laplace transform using Convolution Theorem $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ [6]

c) Show that $\vec{F} = ye^{xy} \cos z \hat{i} + xe^{xy} \cos z \hat{j} - e^{xy} \sin z \hat{k}$ is irrotational vector field. Find ϕ if

$\vec{F} = \nabla\phi$ and also evaluate $\int_P^Q \vec{F} \cdot d\vec{r}$ along a curve joining the points $P(0,0,0)$ and $Q(-1,2, \pi)$. [8]

Q 4 a) Find the Fourier transform of $f(t) = e^{-|t|}$ [6]

b) Show that the function $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on $(-1,1)$ and determine the constant A & B so that functions $f_3(x) = 1 + Ax + Bx^2$ is orthogonal to both $f_1(x)$ and $f_2(x)$ on that interval. [6]



c) Find bilinear transformation which maps the points $z=1, i, -1$ onto the points $w=i, 0, -i$ hence

find the image of $|z| < 1$ on to w plane find invariant points of this transformation [8]

Q 5 a) solve Using the Laplace transform the following system of equations [6]

$$\frac{dX}{dt} = 2X - 3Y, \frac{dY}{dt} = Y - 2X \text{ where } X(0) = 8, Y(0) = 3.$$

b) Find Complex form of the Fourier series for $f(x) = e^{ax}$ in $-\pi < x < \pi$ where 'a' is a

real constant. Hence deduce that $\frac{\pi}{a \sinh a\pi} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n^2 + a^2}$ [6]

c) Verify Green's Theorem in the plane for $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is

the boundary of the region defined by $y = x^2$ and $y = \sqrt{x}$. [8]

Q 6. a) Prove that $J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ [6]

b) Find the map of the line $x-y=1$ by transformation $w = \frac{1}{z}$ [6]

c) Evaluate $\iiint_S \vec{F} \cdot d\vec{s}$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ where S is the region bounded by

$x^2 + y^2 = 4, z = 0, z = 3$ using Gauss divergence theorem. [8]
