



23 NOV 2017

- N.B. 1) Question No. 1 is compulsory.  
 2) Answer **any Three** from remaining  
 3) Figures to the right indicate full marks

1. a) Find Laplace transform of  $f(t) = te^{-3t} \sin t$ . 5
- b) Obtain Complex form of Fourier series of  $f(x) = e^x, -1 < x < 1$  in  $(-1, 1)$ . 5
- c) Does there exist an analytic function whose real part is  $u = k(1 + \cos \theta)$ ? Give justification. 5
- d) The equations of lines of regression are  $3x + 2y = 26$  and  $6x + y = 31$ . Find i) means of  $x$  and  $y$ , ii) coefficient of correlation between  $x$  and  $y$ . 5
2. a) Evaluate  $\int_0^\infty e^t \sin 2t \cos 3t dt$ . 6
- b) Find the image of the square bounded by lines  $x = 0, x = 2, y = 0, y = 2$  in the  $z$ -plane under the transformation  $w = (1 + i)z + 2 - i$ . 6
- c) Obtain Fourier series of  $f(x) = |x|$  in  $(-\pi, \pi)$ . Hence, deduce that - 8  

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$
3. a) Find the inverse Laplace transform of  $F(s) = \frac{s}{(s^2+9)(s^2+4)}$ . 6
- b) Solve  $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$ , with  $u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1 - x)$  taking  $h = 0.1$  for three time steps up to  $t = 1.5$  by Bender - Schmidt method. 6
- c) Using Residue theorem, evaluate
- i)  $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos \theta}$  ii)  $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2}$  8

[TURN OVER]

4. a) Solve by Crank -Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ,  
 $u(0, t) = 0, u(5, t) = 100, u(x, 0) = 20$  taking  $h = 1$  for one-time step. 6

b) Obtain the Taylor's and Laurent series which represent the function

$$f(z) = \frac{z}{(z-1)(z-2)} \text{ in the regions, i) } |z| < 1 \quad \text{ii) } 1 < |z| < 2 \quad 6$$

c) Solve  $(D^2 - 3D + 2)y = 4e^{2t}$  with  $y(0) = -3, y'(0) = 5$  where  $D \equiv \frac{d}{dt}$  8

5. a) Find an analytic function  $f(z) = u + iv$ , if  
 $u = e^{-x}\{(x^2 - y^2) \cos y + 2xy \sin y\}$  6

b) Find the Laplace transform of  $f(t) = t\sqrt{1 + \sin t}$  6

c) Obtain half range Fourier cosine series of  $f(x) = x, 0 < x < 2$ . Using Parseval's identity, deduce that - 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

6. a) If  $f(a) = \oint_C \frac{3z^2 + 7z + 1}{z - a} dz, C: x^2 + y^2 = 4$   
 find the values of  $f(3), f'(1 - i)$  and  $f''(1 - i)$  6

b) Find the coefficient of correlation between height of father and height of son from the following data, 6

Height of father	65	66	67	68	69	71	73
Height of son	67	68	64	68	72	69	70

c) A tightly stretched string with fixed end points  $x = 0$  and  $x = l$ , in the shape defined by  $y = kx(l - x)$  where  $k$  is a constant is released from this position of rest. Find  $y(x, t)$ , the vertical displacement if  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ . 8