



Q. P. Code: 2439  
08 MAY 2018

EXTC/SEM-III/CBCS/Am-III

(3 Hours)

[Total Marks: 80]

- N.B.:** 1) Question No. 1 is Compulsory.  
 2) Answer any **THREE** questions from Q.2 to Q.6.  
 3) Figures to the right indicate full marks.

Q 1. a) Evaluate the Laplace transform of  $\sqrt{1 + \sin t}$  [5]

b) Find directional derivative of  $\phi = 4xz^2 + x^2yz$ , at  $(1, -2, -1)$  in direction of  $2i - j - 2k$  [5]

c) Find orthogonal trajectories of the family of curves  $e^x \cos y - xy = c$ . [5]

d) Obtain half range sine series for  $f(x) = x$ ,  $0 < x < 2$ . [5]

Q 2. a) If  $u + v = e^{2x}(x \cos 2y - y \sin 2y)$  then find analytic function  $f(z)$  by Milne Thomson Method [6]

b) Find the Fourier series for  $f(x) = 9 - x^2$ ,  $-3 \leq x \leq 3$  [6]

c) Find the Laplace transform of the following

- i)  $L[t\sqrt{1 + \sin t}]$       ii)  $L\left[\frac{\sinh 2t}{t}\right]$  [8]

Q 3. a) Using Convolution theorem, find Inverse Laplace of  $\frac{3}{(s^2 + 4)^2}$  [6]

b) Prove that  $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left[ \frac{3}{x} \sin x + \frac{(3-x^2)}{x^2} \cos x \right]$ . [6]

c) Find Fourier series for  $f(x) = (\pi - x)^2$  in  $0 \leq x \leq 2\pi$ . Hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$
 [8]

Q 4 a) Find the Fourier transform of  $f(t) = e^{-|t|}$  [6]

b) Show that the function  $f_1(x) = 1, f_2(x) = x$  are orthogonal on  $(-1, 1)$  and determine the constant A & B so that functions  $f_3(x) = 1 + Ax + Bx^2$  is orthogonal to both  $f_1(x)$  and  $f_2(x)$  on that interval. [6]

c) Find bilinear transformation which maps the points  $z=1, i, -1$  onto the points  $w=i, 0, -i$  hence





find the image of  $|z| < 1$  on to w plane find invariant points of this transformation [8]

Q.5 a) Solve using Laplace Transform  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}$  given  $y(0) = 4$  and  $y'(0) = 2$ . [6]

b) Find Complex form of the Fourier series for  $f(x) = e^{ax}$  in  $-\pi < x < \pi$  where 'a' is a real constant. Hence deduce that  $\frac{\pi}{a \sinh a\pi} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n^2 + a^2}$  [6]

c) Verify Green's Theorem in the plane for  $\int_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$  where C is the boundary of the region defined by  $y = x^2$  and  $y = \sqrt{x}$  [8]

Q.6. a) Prove that  $J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$  [6]

b) Find the map of the line  $x-y=1$  by transformation  $w = \frac{1}{z}$  [6]

c) Evaluate  $\iiint_S \vec{F} \cdot d\vec{s}$  where  $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$  where S is the region bounded by  $x^2 + y^2 = 4, z = 0, z = 3$  using Gauss divergence theorem. [8]

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