



# Discrete Structures

JUN 19

Computer Engineering (Semester 3)

**Total marks: 80**

**Total time: 3 Hours**

## INSTRUCTIONS

(1) Question 1 is compulsory.

(2) Attempt any **three** from the remaining questions.

(3) Draw neat diagrams wherever necessary.

**1.a.** Prove using Mathematical Induction  $2+5+8+\dots+(3n-1)=n(3n+1)/2$  (5 marks)

**1.b.** Find the generating function for the following finite sequences  
i) 1,2,3,4,... ii) 2,2,2,2,2 (5 marks)

**1.c.** Let  $A = \{1, 4, 7, 13\}$  and  $R = \{(1,4), (4,7), (7,4), (1,13)\}$   
Find Transitive Closure using Warshall's Algorithm (5 marks)

**1.d.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ , where  $f(x) = 2x - 1$  and  
 $f^{-1}(x) = (x+1)/2$  (05M) Find  $(f \circ f^{-1})(x)$  (5 marks)

**2.a.** Define Lattice. Check if the following diagram is a lattice or not.



(4 marks)

**2.b.** Prove that set  $G = \{1,2,3,4,5,6\}$  is a finite abelian group of order 6  
with respect to multiplication module 7 (8 marks)

**2.c.** A travel company surveyed its travellers, to learn how much of their travel is taken with an Air plane, a Train or a Car. The following data is known; make a complete Venn Diagram with all the data. The number of people who flew was 1307. The number of people who both flew and used a train was 602. The people who used all three were 398 in number. Those who flew but didn't drive came to a total of 599. Those who drove but did not use a train totaled 1097. There were 610 people who used both trains and cars. The number of people who used either a car or a train or both was 2050. Lastly, 421 people used none of these Find out how many people drove but used neither a train nor an airplane, and also, how many people were in the entire survey. (8 marks)



**3.a.** . (08 M) Q.3 a) Prove  $\neg(p \vee (\neg p \wedge q)) \equiv \neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q \equiv \neg(p \vee q)$  are logically equivalent by developing a series of logical equivalences. (4 marks)

**3.b.** Consider the (3,5) group encoding function defined by  $e(000)=0000$  ,  $e(001)=00110$

$e(010)=01001$  ,  $e(011)= 01111$

$e(100)=10011$  ,  $e(101)=10101$

$e(110)=11010$  ,  $e(111)=11000$  (8 marks)

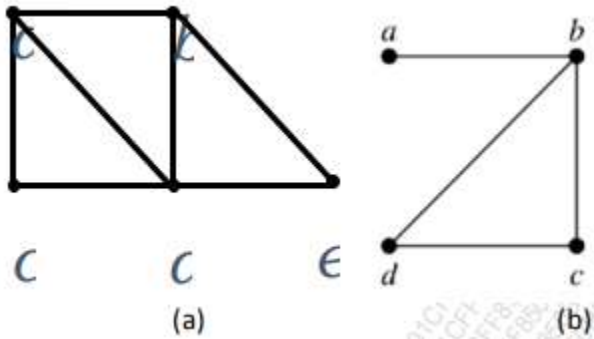
**3.c.** Mention all the elements of set  $D_{36}$  also specify  $R$  on  $D_{36}$  as  $aRb$  if  $a \mid b$ . Mention Domain and Range of  $R$ . Explain if the relation is Equivalence Relation or a Partially Ordered Relation. If it is a Partially Ordered Relation, draw its Hasse Diagram.  $D_{36}$  (8 marks)

**4.a.** Explain Extended pigeonhole Principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. (4 marks)

**4.b.** Define Euler Path and Hamiltonian Path.

i) Determine Euler Cycle and path in graph shown in (a)

ii) Determine Hamiltonian Cycle and path in graph shown in (b)



(8 marks)

**4.c.** In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there? (8 marks)

**5.a.** Let  $G$  be a group. Prove that the identity element  $e$  is unique. (4 marks)



**5.b.** A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, NOT replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen? (4 marks)

**5.c.** Let A be a set of integers, let R be a relation on AXA defined by (a,b) R (c,d) if and only if  $a+d=b+c$ . Prove that R is an equivalence Relation. (8 marks)

**6.a.** Define reflexive closure and symmetric closure of a relation.

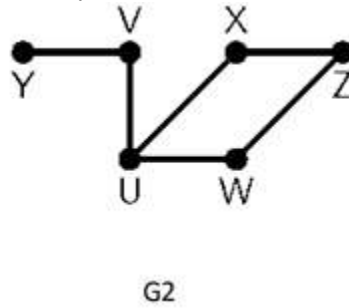
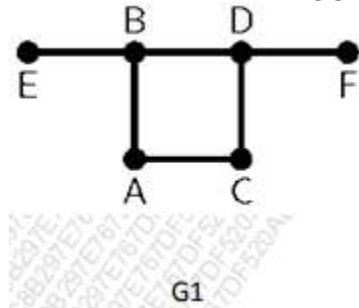
Also find reflexive and symmetric closure of R.

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)\}$$

b) Let  $H = |10110001101001100110001100011111100010001|$  Be a parity check matrix. Determine the group  $eHB_3 \rightarrow B_6eHB_3 \rightarrow B_6$  (8 marks)

**6.c.** Determine if following graphs  $G_1$  and  $G_2$  are isomorphic or not.



(8 marks)