Discrete Mathematics

DEC 18

Computer Engineering (Semester 3)

Total marks: 80
Total time: 3 Hours

INSTRUCTIONS
(1) Question 1 is compulsory.
*(2) Attempt any **three** from the remaining questions.*
(3) Draw neat diagrams wherever necessary.

1.a. Two dice are rolled. find the probability that the sum is
i) Equal to 1

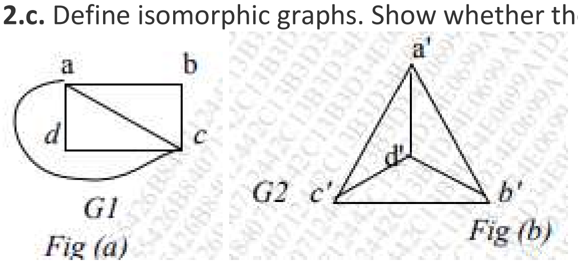
ii) Equal to 4

ii) less than 13 (6 marks)

1.b. Use the laws of logic to show that
[(p⇒q)[(p⇒q) ^ ~q] ⇒⇒ ~p is a tautology (6 marks)
1.c. Determine the matrix of the partial order of divisibility on the set A.
Draw the Hasse diagram of the poset. Indicate those which are chains.
i) A = {1,2,3,5,6,10,15,30}

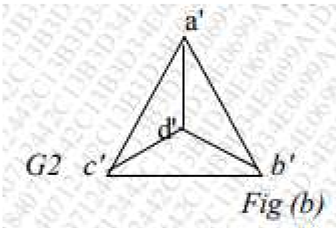
ii) B = {3,6,12,36,72} (8 marks)

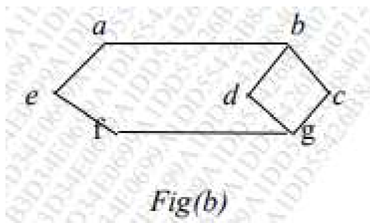
2.a. Find the complement of each element in D42D42 (6 marks)
2.b. Let Q be the set of positive rational numbers which can be expressed
in the form of 2a2a 3b3b , where a and b are integers.
Prove that algebraic structure (Q , .) is a group.
Where . is multiplication operation. (6 marks)
2.c. Define isomorphic graphs. Show whether the following are isomorphic or not



(8 marks)

3.a. Determine which of the following graph contains an Eulerian or Hamiltonian circuit.





(6 marks)

3.b. For all sets A,X and Y show that
A x (X ∩∩ Y) = (A x X) ∩∩ (A x Y) (6marks)
3.c. Let f(x) = x + 2, g(x) = x - 2, and h(x) = 3x for x ∈∈ R, where
R = set of real numbers .
Find (g , f) , (f , g) , (f , f) , (g , g) , (f , h) , (h , g) , (h , f) , (f , h , g) (8 marks)

4.a. Let R is a binary relation. Let S = {(a,b), (a , c) ∈∈ R and
(c , b) ∈∈ R for some c} Show that if R is an equivalence relation
then S is also an equivalence relation. (6 marks)
4.b. Determine the generating function of the numeric function arar ,
Where
i) arar = 3r3r + 4r+14r+1 , r ≥≥ 0
ii) arar = 5, r ≥≥ 0 (6 marks)
4.c. Consider the (3,6) encoding function e : B3⇒B6B3⇒B6 defined by
e(000) = 000000 e(001) = 001100 e(010) = 010011 e(011) = 011111

e(100) = 100101 e(101) = 101001 e(110) = 110110 e(111) = 111010

Decode the following words relative to a maximum likelihood decoding function.

i) 000101 ii) 010101 (8 marks)

5.a. Determine the number of positive integers n where 1 ≤≤ n ≤≤ 100 and
n is not divisible by 2,3, or 5. (6 marks)
5.b. Use mathematical induction to show that
1+5+9+..............+ (4n-3) = n (2n-1) (6 marks)

5.c. Find the greatest lower bound and least upper bound of the set
{3 , 9, 12} and {1 , 2, 4 , 5, 10} if they exists in the poset (z+z+ , /).
where / is the relation of divisibility. (8 marks)

6.a. Let A = {1 , 2, 3 , 4} and Let
R = {(1 ,1) (1, 2) (1 , 4) (2 , 4) (3 , 1) (3 , 2) (4 , 2) (4 , 3) (4 , 4)} .
Find transitive closure by Warshall's algorithm. (6 marks)

6.b. Let H = {[0]6,[3]6[0]6,[3]6} find the left and right cosets in group Z6Z6.
Is H a normal subgroup of closure by Warshall's algorithm. (6 marks)

6.c. Find the complete solution of the recurrence relation
an+2an−1an+2an−1 = n + 3 for n ≥≥ 1 and with a0=3a0=3 (8 marks)