LAST MOMENT TUITIONS

Applied Maths -I May 18

First Year (Semester 1)

Total marks: 80
Total time: 3 Hours

INSTRUCTIONS
(1) Question 1 is compulsory.
*(2) Attempt any **three** from the remaining questions.*
(3) Draw neat diagrams wherever necessary.

1(a) If $\tan\frac{x}{2}=\tanh\frac{u}{2}$, show that $u=\log[\tan(\frac{\pi}{4}+\frac{x}{2})]$ (3 marks)

1(b) Prove that the following matrix is orthogonal & hence find A^{-1}

$$A=\frac{1}{3}\cdot\begin{matrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{matrix}$$ (3 marks)

1(c) State eulers theorem on homogeneous function of two variables & if $u=\frac{x+y}{x^2+y^2}$ then evaluate $x\frac{\delta u}{\delta x}+y\frac{\delta u}{\delta y}$. (3 marks)

1(d) If $u=r^2\cos 2\theta, v=r^2\sin 2\theta$. Find $\frac{\delta(u,v)}{\delta(r,\theta)}$ (3 marks)

1(e) Find the n^{th} derivative of cos5x.cos3x (4 marks)

1(f) Evaluate: $\lim_{x\to 0}\left(\frac{2x+1}{x+1}\right)^{\frac{1}{x}}$ (4 marks)

2(a) Solve $x^4-x^3+x^2-x+1=0$ (6 marks)

2(b) If $y=e^{\tan^{-1}x}$. Prove that $(1+x^2)y_{n+2}+[2(n+1)x-1]y_{n+1}+n(n+1)y_n=0$. (6 marks)

2(c) Examine the function $f(x,y)=xy(3-x-y)$ for extremes values & also find maximum and minimum values of f(x,y) (8 marks)

3(a) Investigate for what values of λ and μ the equations x+y+z = 6; x+2y+3z = 10; x+2y+λz = μ have

- no solution
- a unique solution
- infinite number of solution (6 marks)

3(b) If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, show that $x^2\frac{\delta u}{\delta y}+y^2\frac{\delta u}{\delta y}+z^2\frac{\delta u}{\delta y}=0$ (6 marks)

3(c) prove that $\log\left(\frac{a+ib}{a-ib}\right)=2i\tan^{-1}\frac{b}{a}$ & $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]=\frac{a^2-b^2}{a^2+b^2}$ (8 marks)

4(a) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=\frac{-\sin u\cos 2u}{4\cos^2 u}$ (6 marks)

4(b) using encoding matrix $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$; encode & decode the message "ALL IS WELL" (6 marks)

4(c) Solve the following equations by Gauss Seidal method:

$10x_1+x_2+x_3=12$

$2x_1+10x_2+x_3=13$

$2x_1+2x_2+10x_3=14$ (8 marks)

5(a) If $u=e^{xyz}f(\frac{xy}{z})$ where, $f(\frac{xy}{z})$ is an arbitrary function of $\frac{xy}{z}$, prove that:

$$x\frac{\delta u}{\delta x}+z\frac{\delta u}{\delta z}=y\frac{\delta y}{\delta x}+z\frac{\delta u}{\delta z}=2xyz.u$$ (6 marks)

5(b) prove that $\sin^5\theta=\frac{1}{16}(\sin 5\theta-5\sin 3\theta+10\sin\theta)$ (6 marks)

5(c)

1)Prove that $\log(\sec x)=\frac{1}{2}x^2+\frac{1}{12}x^4+$.........................

2)Expand $(2x^3+7x^2+x-1)$ in powers of $(x-2)$ (8 marks)

6(a) Prove that $\sin^{-1}(\operatorname{cosec}\theta)=\frac{\pi}{2}+i\log(\cot\frac{\theta}{2})$ (6 marks)

6(b) Find the non - singular matrices P&Q such that $A = \begin{matrix}1 & 2 & 3 & 2\\2 & 3 & 5 & 1\\1 & 3 & 4 & 5\end{matrix}$ is reduced to normal form.

Also, find its rank. (6 marks)

6(c) Obtain the root of $x^3-x-1=0$ by Regula Falsi Method (Take three iterations) (8 marks)