Applied Maths -I May 17

First Year (Semester 1)

Total marks: 80
Total time: 3 Hours

INSTRUCTIONS
(1) Question 1 is compulsory.
*(2) Attempt any **three** from the remaining questions.*
(3) Draw neat diagrams wherever necessary.

1(a) Prove that $\tanh^{-1}(\sin\theta)=\cosh^{-1}(\sec\theta)$ (3 marks)

1(b) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary (3 marks)

1(c) If x=uv & $y=\frac{u}{v}$ prove that $JJ'=1$ (3 marks)

1(d) If $Z=\tan^{-1}(\frac{x}{y})$ where $x=2t, y=1-t^2$, prove that $\frac{dZ}{dt}=\frac{2}{1+t^2}$ (3 marks)

1(e) Find the n^{th} derivative of (cos5x.cos3x.cosx) (4 marks)

1(f) Evaluate $\lim_{x\to 0}(x)^{\frac{1}{x+1}}$ (4 marks)

2(a) Find all values of $(1+i)^{\frac{1}{3}}$ & show that their continued product is (1+i) (6 marks)

2(b) Find the non-singular matrices P&Q such that PAQ is normal from where

$$A=\begin{matrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{matrix}$$ (6 marks)

2(c) Find the maximum and minimum values of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72$ (8 marks)

3(a) If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$ show that

x^2\frac{\delta u}{\delta z} + y^2\frac{\delta u}{\delta y} + z^2\frac{\delta u}{\delta z} = 0 (6 marks)

3(b) using Encoding matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ encode & decode the message "MUMBAI" (6 marks)

3(c) Prove that $\log[\tan(\frac{\pi}{4}+\frac{ix}{2})]=i\tan^{-1}(\sinh x)$ (8 marks)

4(a) Obtain tan5θ in terms of tanθ & show that $1-10\tan^2\frac{\pi}{4}+5\tan^4\frac{\pi}{10}=10$ (6 marks)

4(b) If $y=e^{\tan^{-1}x}$. Prove that

$(1+x^2)y_{n+2}+[2(n+1)x-1]y_{n+1}+n(n+1)y_n=0$ (6 marks)

4(c)

- Express $(2x^3+3x^2-8x+7)$ in terms of (x-2) using Taylors theorem
- prove that $\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}$ — — — — — — — — — — — — (8 marks)

5(a) If $z=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$, prove that $\frac{\delta^2 z}{\delta x\delta y}=\frac{x^2-y^2}{x^2+y^2}$ (6 marks)

5(b) Investigate the values of λ & μ the equation

2x+3y+5z=9
7x+3y−2z=8
2x+3y+\lambdaz=\mu$$
1) have no solution

2) a unique solution

3) an infinite no of solution (6 marks)

5(c) Using Newton Raphson method, find approximate root of $x^3-2x-5=0$ (correct to three places of decimals). (8 marks)

6(a) Find tanhx if 5sinhx - coshx = 5 (6 marks)

6(b) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$,

- $xu_x+yu_y=\frac{x}{y}\tan u$
- prove that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=\frac{-\sin u\cos 2u}{4\cos^3 u}$ (6 marks)

6(c) Solve the following equations by Gauss Seidal method:

20x+y+2z=17
3x+20y−z=−18
2x+3y+20z=25 (8 marks)