LAST MOMENT TUITIONS

Applied Maths -I Dec 17

First Year (Semester 1)

Total marks: 80
Total time: 3 Hours

INSTRUCTIONS
(1) Question 1 is compulsory.
*(2) Attempt any **three** from the remaining questions.*
(3) Draw neat diagrams wherever necessary.

1(a) Separate into a real part and imaginary part of $\cos^{-1}\left(\frac{3i}{4}\right)$ (3 marks)

1(b) Show that the matrix A is unitary where A= $\begin{matrix} \alpha+i\gamma & -\beta+i\gamma \\ \beta+i\delta & \alpha-i\gamma \end{matrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2=1$ (3 marks)

1(c) If z=tan(y+ax)+$(y-ax)^{\frac{3}{2}}$ then show that $\frac{\delta^2 z}{\delta x^2}$=$a^2\frac{\delta^2 z}{\delta y^2}$ (3 marks)

1(d) If x=uv,y=$\frac{u}{v}$ Prove that $JJ^{/}=1$ (3 marks)

1(e) Find the n^{th} derivative of $\frac{x^3}{(x+1)\quad(x-2)}$ (4 marks)

1(e) Using the matrix A=$\begin{matrix} -1 & 2 \\ -1 & 1 \end{matrix}$decode the message matrix

C=$\begin{matrix} 4 & 11 & 12 & -2 \\ -4 & 4 & 9 & -2 \end{matrix}$ (4 marks)

2(a) If $\sin^4\vartheta\cos^3\theta = a\cos\theta + b\cos 3\theta + c\cos 7\theta$ then find a, b, c, d. (6 marks)

2(b) Using Newtons Raphson method Solve 3x - cosx - 1 = 0 Correct to 3 decimal places. (6 marks)

2(c) Find the stationary points of the function $x^3+3xy^2-3x^2-3y^2+4$ & also find maximum and minimum values of the function. (8 marks)

3(a) Show that $x\csc x = 1+\frac{x^2}{6}+\frac{7}{360}x^4$+...................... (6 marks)

3(b) Reduce matrix to PAQ normal form and find 2 non-singular matrices P&Q

$\begin{matrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{matrix}$ (6 marks)

3(c) If y=cos(m$\sin^{-1}$x) prove that $(1-x^2)$yn+2−(2n+1)xyn+1+(m^2-n^2)yn=0 (8 marks)

4(a) State and prove Euler's theorem for three Variables. (6 marks)

4(b) Show that all the roots of $(x+1)^6+(x-1)^6=0$ are given by $-i\cot\frac{(2k+1)\pi}{12}$

where k = 0, 1, 2, 3, 4, 5 (6 marks)

4(c) Show that the equations

-2x + y + z = a

x -2y + z = b

x + y -2z = c

have no solutions unless a+b+c = 0 in which case they have infinitely many solutions.

Find these solutions when a=1, b=1, c=-2 (8 marks)

5(a) If z=f(x,y),x=rcosθ and y=rsinθ

$\left(\frac{\delta z}{\delta x}\right)^2+\left(\frac{\delta z}{\delta r}\right)^2=\left(\frac{\delta z}{\delta r}\right)^2+\frac{1}{r^2}\left(\frac{\delta z}{\delta \theta}\right)^2$ (6 marks)

5(b) If coshx=secθ prove that

- x=log(secθ+tanθ)
- $\theta=\frac{\pi}{2}-2\tan^{-1}(e^{-x})$ (6 marks)

5(c) Solve by Gauss Jacobi Iteration method

5x - y + z = 10

2x + 4y = 12

x + y + 5z = -1 (8 marks)

6(a) prove that $\cos^{-1}[\tanh(\log x)]=\pi-2\left(x-\frac{x^3}{3}+\frac{x^5}{5}-------\right)$ (6 marks)

6(b) If $y=e^{2x}\sin\frac{x}{2}\cos x 2\sin 3x$ .Find y_n (6 marks)

6(c) Evaluate $\lim_{x\to 0}(\cot x)^{\sin x}$

Prove that $\log\left[\frac{\sin x+iy}{\sin x-iy}\right]=2i\tan^{-1}(\cot x\tan hy)$ (8 marks)