LAST MOMENT TUITIONS

Applied Maths -I Dec 16

First Year (Semester 1)

Total marks: 80
Total time: 3 Hours

INSTRUCTIONS
(1) Question 1 is compulsory.
*(2) Attempt any **three** from the remaining questions.*
(3) Draw neat diagrams wherever necessary.

1(a) If cosα coshβ=$\frac{x}{2}$,sinα sinhβ=$\frac{y}{2}$,/ Prove that sec(α−iβ)+sec(α+iβ)=$\frac{4x}{x^2+y^2}$/ (3 marks)

1(b) If z=log(e^x+e^y)/, show that rt-s^2=0, where r=$\frac{\partial^2 z}{\partial x^2}$,t=$\frac{\partial^2 z}{\partial y^2}$,s=$\frac{\partial^2 z}{\partial x \partial y}$ / (3 marks)

1(c) If x = u v, (3 marks)

$$y = \frac{u+v}{u-v}$$

Find

$$\frac{\partial(u,v)}{\partial(x,y)}$$

1(d) If y=$2^x\sin^2 x\cos x$/ find y_n (3 marks)

1(e) Express the matrix A=$\begin{bmatrix} 1 & 0 & 5 & 3 \\ -2 & 1 & 6 & 1 \\ 3 & 2 & 7 & 1 \\ 4 & -4 & 2 & 0 \end{bmatrix}$as the sum of symmetric and skew- symmetric matrices.

(4 marks)

1(f) Evaluate $\lim_{x\to 0}\frac{e^2-(1+x)^2}{x\log(1+x)}$ (4 marks)

2(a) Show that the roots of x^5=1 can be written as 1, α, α^2, α^3, α^4.
Hence show that (1−α)(1−α^2)(1−α^3)(1−α^4)=5 (6 marks)

2(b) Reduce the following matrix to its normal from and hence find its rank (6 marks)

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

2(c) Solve the following system of equations by Gauss-Seidel Iterative Method upto four interactions.
4x-2y-z=40

x-6y+2z=-28
x-2y+12z = -86 (8 marks)

3(a) Investigate for what values of 'λ' and 'μ' the system of equations x+y+z=6x+2y+3z=10x+2y+λz/ has
i) no solution
ii) a unique solution
iii) an infinite no. of solutions. (6 marks)

3(b) If $u=x^2+y^2+z^2$/, where $x=e^t, y=e^t \sin t, z=e^t \cos t$/

Prove that $\frac{du}{dt}=4e^{2t}$ (6 marks)

3(c)(i) Show that $\sin(e^x-1)=x+\frac{x^2}{2}-\frac{5x^4}{24}+\ldots\ldots$ (4 marks)
3(c)(ii) Expand $2x^3+7x^2+x-6$ in power of x-2 (4 marks)

4(a) If x=u+v+w, y = uv+vw+uw, z=uvw and ϕ is a function of x,y and z. (6 marks)
Prove that $x\frac{\partial\phi}{\partial x}+2y\frac{\partial\phi}{\partial y}+3z\frac{\partial\phi}{\partial z}=u\frac{\partial\phi}{\partial u}+v\frac{\partial\phi}{\partial v}+\frac{\partial\phi}{\partial w}$

4(b) if tan(θ+iφ)=tanα+isecα/, Prove that
i) $e^{2\varphi}=\cot\frac{\alpha}{2}$
ii) $2\theta=n\pi+\frac{\pi}{2}+\alpha$ (6 marks)

4(c) Find the root of the equation $x^4+x^3+7x^2-x+5=0$ which lies between 2 and 2.1 correct to three places of decimals using Regula Falsi Method. (8 marks)

5(a) If $y=\left(x+\sqrt{x^2-1}\right)^m$ /, (6 marks)
Prove That $(x^2-2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$

5(b) Using the encoding matrix $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$/, encode and decode the message I* LOVE*MUMBAI* (6 marks)

5(c)(i) Consulting only principal values separate into real and imaginary parts
i^\log \left (1+i \right) (4 marks)
5(c)(ii) Show that (4 marks)
$i\log\left(\frac{x-1}{x+1}\right)=\pi-2\tan^{-1}x$

6(a) Using De Moivre's theorem prove that

$$\cos^6\theta - \sin^6\theta = \frac{1}{16}(\cos 6\theta + 15\cos 2\theta)$$ (6 marks)

6(b) If $u = \sin^{-1}\left(\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}}\right)^{\frac{1}{2}}$ /, (6 marks)

Prove that $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(tan^2 u + 13)$$

6(c) Discuss the maxima and minima of

$$f(x,y) = x^3y^2(1-x-y)$$ (8 marks)