



Applied Mathematics 2 - May 18

First Year Engineering (Semester 2)

Total marks: 80

Total time: 3 Hours

INSTRUCTIONS

(1) Question 1 is compulsory.

(2) Attempt any **three** from the remaining questions.

(3) Draw neat diagrams wherever necessary.

1.a. Evaluate $\int_0^{\infty} 5^{-4x^2} dx$ (3 marks)

1.b. Solve $\frac{dy}{dx} = xy$ with the help of Euler's method, given that $y(0) = 1$ and find y when $x = 0.3$ ($h = 0.1$) (3 marks)

1.c. Evaluate $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$ (3 marks)

1.d. Evaluate $\int_0^1 \sqrt{\sqrt{x} - x} dx$ (3 marks)

1.e. Solve $(1 + \log xy)dx + (1 + \frac{x}{y})dy = 0$ (4 marks)

1.f. Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx dy}{1+x^2+y^2}$ (4 marks)

2.a. Solve $xy(1+xy^2)\frac{dy}{dx} = 1$ (6 marks)

2.b. Find the area inside the circle $r = a \sin \theta$ and outside the cardioid $r = a(1 + \cos \theta)$ (6 marks)

2.c. Apply Runge-Kutta Method of fourth order to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ when $y = 1$ at $x = 0$ with step size $h = 0.2$ (8 marks)

3.a. Show that the length of the curve $9ay^2 = x(x-3a)^2$ is $4\sqrt{3}a$ (6 marks)

3.b. Change the order of the integration of $\int_0^1 \int_{-\sqrt{2y-y^2}}^{1+\sqrt{1-y^2}} f(x, y) dx dy$ (6 marks)

3.c. Find the volume of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ (8 marks)



4.a. Show that $\int_0^1 \frac{x-1}{\log x} dx = \log(a+1)$ (6 marks)

4.b. If Y satisfies the equation $\frac{dy}{dx} = x^2y - 1$ with $x_0=0, y_0=1$, using Taylor's series method find y at $x = 0.1$ (take $h = 0.1$) (6 marks)

4.c. Find the value of the integral $\int_0^1 \frac{x^2}{1+x^2} dx$ using

(i) Trapezoidal rule

(ii) Simpsons 1/3rd rule

(iii) Simpsons 3/8th rule. (8 marks)

5.a. Solve $(y-xy^2)dx - (x+x^2y)dy = 0$ (6 marks)

5.b. Evaluate $\iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}} dx dy dz$ over the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (6 marks)

5.c. Evaluate $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 6x$. (8 marks)

6.a. A resistance of 100 ohms and inductance of 0.5 henneries are connected in series with a battery of 20 volts. Find the current at any instant if the relation between L, R, E is $L \frac{di}{dt} + Ri = E$ (6 marks)

6.b. Solve by variation parameter method $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ (6 marks)

6.c. Evaluate $\iint xy(x-y) dx dy$ over the region bounded by $xy = 4, y = 0, x = 1$ and $x = 4$. (8 marks)