Applied Mathematics 2 - Dec 2016

First Year Engineering (Semester 2)

TOTAL MARKS: 80
TOTAL TIME: 3 HOURS
*(1)**Question 1 is compulsory**.*
*(2) Attempt **any three** from the remaining questions.*
(3) Assume data if required.
(4) Figures to the right indicate full marks.

1(a) Solve

$$\left[\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right]dx+\left(\frac{2xy}{x^2+y^2}\right)dy=0$$ (4 marks)

1(b) Solve

$$(D^4+2D^2+1)y=0$$ (3 marks)

1(c) Evaluate

$$\int_0^\infty e^{-x^5}\,dx$$ (3 marks)

1(d) Express the following integral in polar co-ordinates:

$$\int_0^{\frac{a}{\sqrt{2}}}\int_y^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy$$ (4 marks)

1(e) Prove that $E=1+\Delta=e^{hD}$ (3 marks)

1(f) Evaluate $1\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi} \cos(x+y)\,dx\,dy$ (3 marks)

2(a) Solve $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log x)^2$

(6 marks)

2(b) Change the order of integration and evaluate

$$1= \int_0^2 \int_{\sqrt{2y}}^{2} \frac{x^2\, dx\, dy}{\sqrt{(x^4-4y^2)}}$$ (6 marks)

2(c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+asin^2x}$ and duduce that

$$\int_0^{\frac{\pi}{2}} \frac{sin^2x dx}{(3+asin^2x)^2} = \frac{\pi\sqrt{3}}{96}$$ (6 marks)

3(a) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+2}\, dx\, dy\, dz$ (6 marks)

3(b) If mass per unit area varies as the square of the ordinate of a point, find the mass of a lamina bounded by the cycloid y=a (1−cosθ), x=a (θ+sinθ)/ and the ordinates from the two cups and the tangents at the vertex.

(6 marks)

3(c) Solve $(2x+1)^2\frac{d^2y}{dx^2} - 6(2x+1)\frac{dy}{dx} + 16y = 8(2x+1)^2$ (8 marks)

4(a) Show that the length of the are of the parabola y^2=4ax cut off by the line

3y=8x is a[log2+$\frac{15}{16}$] (6 marks)

4(b) Solve $(2x+1)^2\frac{d^2y}{dx^2} - 6(2x+1)\frac{dy}{dx} + 16y = 8(2x+1)^2$ (6 marks)

4(c) Using fourth order Runge-Kutta method, find u (0, 4) of the initial value problem u'=2tu^2, u (0)=1 take h =0.2. (8 marks)

5(a) Use method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}x^2$$ (6 marks)

5(b) Using Taylor's series method, obtain the solutions of

$$\frac{dy}{dx} 3x + y^2, y(0) = 1$$

Find the value of y for x = 0.1 correct to four decimal places (6 marks)

5(c) Find the value of the integral $\int_0^1 \frac{x^2}{1+x^3} dx$ by taking h=0.2, using
i) Trapezoidal Rule ii) Simpson's 1/3 Rule. Compare errors with the exact value of the integral (8 marks)

6(a) A condenser of capacitance C is charged through a resistance R by a steady voltage. The charge Q satisfies the DE R$\frac{dQ}{dt}$+$\frac{Q}{C}$ =V/ If the plate is chargeless find the charge and the current at time't' (6 marks)

6(b) Evaluate $\iint \frac{(x^2+y^2)^2}{x^2y^2}$dxdy/ over the region common to $x^2+y^2-ax=0$/and $x^2+y^2-by=0$, a>0, b>0? /

(6 marks)

6(c) Find the volume common to the right circular cylinder

$$x^2+y^2=a^2 \text{and } x^2+z^2=a^2$$

(8 marks)