



Applied Mathematics 2 - Dec 18

First Year Engineering (Semester 2)

Total marks: 80

Total time: 3 Hours

INSTRUCTIONS

(1) Question 1 is compulsory.

(2) Attempt any **three** from the remaining questions.

(3) Draw neat diagrams wherever necessary.

1.a. Evaluate $\int_0^{\infty} \frac{e^{-x^3}}{\sqrt{x}} dx$ (3 marks)

1.b. Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 2$ (3 marks)

1.c. Solve $(D^2 + D)y = e^{4x}$ (3 marks)

1.d. Evaluate $\int_0^1 \int_{x^2}^x xy(x+y) dy dx$ (3 marks)

1.e. Solve $(4x + 3y - 4) dx + (3x - 7y - 3) dy = 0$ (4 marks)

1.f. Solve $\frac{dy}{dx} = 1 + xy$ with initial condition.

$x_0 = 0, y_0 = 0.2$ by Taylor's series method. Find the approximate value of y for $x = 0.4$ (step size 0.4) (4 marks)

2.a. Solve $\frac{dy}{dx} - 16y = x^2 e^{3x} + e^{2x} - \cos 3x + 2^x$ (6 marks)

2.b. Show that $\int_0^{\pi} \frac{\log(1 + a \cos x)}{\cos x} \pi \sin^{-1} a \, 0 \leq a \leq 1$ (6 marks)

2.c. Change the order of integration and evaluate $\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} dx dy$ (8 marks)

3.a. Evaluate $\iiint (x+y+z) dx dy dz$ over the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$ (6 marks)

3.b. Find the mass of the lamina bounded by the curves $y = x^2 - 3x$ and $y = 2x$ if the density of the lamina at any point is given by $\frac{24}{25}xy$ (6 marks)

3.c. Solve $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 3y = \frac{\log x \cdot \cos(\log x)}{x}$ (8 marks)



4.a. Find by double integration the area bounded by the parabola $y^2=4x$ and the line $y = 2x - 4$ (6 marks)

4.b. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ (6 marks)

4.c. Solve $\frac{dy}{dx} = x^3 + y$ with initial conditions $y(0) = 2$ at $x = 0.2$ in steps of $h = 0.1$ by Runge Kutta method of fourth order. (8 marks)

5.a. Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$ and find the value of $\beta(\frac{9}{2}, \frac{1}{2})$ (6 marks)

5.b. In a circuit containing inductance L , resistance R , and voltage E , the current I is given by $L \frac{di}{dt} + Ri = E$ find the current i at time t if at $t = 0$, $i = 0$ and L, R, E are constants. (6 marks)

5.c. Evaluate $\int_0^6 \frac{dx}{1+3x}$ by using

i) Trapezoidal

ii) Simpson's (1/3)rd and

iii) Simpsons (3/8)th rule

(8 marks)

6.a. Find the volume bounded by the paraboloid.

$x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$ (6 marks)

6.b. Change to polar co-ordinates and evaluate.

$\int_0^1 \int_0^x (x + y) \, dy \, dx$ (6 marks)

6.c. Solve by method of variation of parameters.

$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ (8 marks)